Adaptively chosen sampling points: Analysis of target data dependent greedy kernel algorithms

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Abstract

We consider the reconstruction of functions in reproducing kernel Hilbert spaces (RKHS) based on function values. For this, data-dependent greedy algorithms in kernel spaces are known to provide fast converging interpolants, while being extremely easy to implement and efficient to run. Despite this experimental evidence, no detailed theory has yet been presented.

In this work we fill this gap by first defining a new scale of greedy algorithms for interpolation that comprises all the existing ones in a unique analysis, where the degree of dependency of the selection criterion on the functional data is measured by a real parameter. We then prove new convergence rates where this degree is taken into account and we show that, possibly up to a logarithmic factor, data dependent selection strategies provide faster convergence.

In particular, for the first time we obtain convergence rates for adaptive interpolation that are faster than the ones given by uniform points or randomly selected points, without the need of any special assumption on the target function.

Especially those rates do not depend on the dimensionality of the domain, i.e. the adaptivity of the sampling points breaks the curse of dimensionality.

Joint work with: Gabriele Santin, Bernard Haasdonk.

References

[1] T. Wenzel, G. Santin, and B. Haasdonk. Analysis of target data-dependent greedy kernel algorithms: Convergence rates for f-, $f \cdot P$- and f/P-greedy. *ArXiv*, (2105.07411), 2021. Accepted for publication in Constructive Approximation.

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