From Physics to Noncommutative Geometry and Back

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Modern physical theories draw handfuls of structures from differential geometry. This sounds fairly intuitive when one thinks of general relativity (aka "spacetime geometry") or classical mechanics, but how about quantum theory? While the phase space of a classical system — spanned by the available positions and momenta of particles — is a space indeed, for a quantum system this is no longer the case. The positions and momenta of quantum particles naturally determine a noncommutative algebra rather than a space.

Noncommutative geometry arose from the quest to understand the geometry of the quantum. Pondered already by Werner Heisenberg — one of the founding fathers of quantum mechanics — in 1930s, it acquired a more concrete shape only at the end of the previous century. In essence, noncommutative geometry is concerned with seeking analogues of differential-geometric structures in the realm of complex noncommutative algebras. Within the lecture we shall focus on noncommutative geometry as put forward and developed by Alain Connes.

We shall start the lecture with reviewing the major physical and mathematical motivations behind noncommutative geometry. Then, we will get familiar with the basic structure — a spectral triple — through definitions and examples. Next, we will encounter the spectral action principle, its implementation and tools for computations. The latter would bring us back to physics. We shall discuss some modern applications of noncommutative geometry in particles physics, as well as the ambitious challenge to build a consistent theory of quantum spacetimes.

Recommended literature:

[1] A. Connes. Noncommutative Geometry. Academic Press, 1994, https://alainconnes.org/wp-content/uploads/book94bigpdf.pdf

[2] W. D. van Suijlekom. Noncommutative Geometry and Particle Physics. Mathematical Physics Studies. Springer, 2015, http://www.waltervansuijlekom.nl/wp-content/uploads/2016/06/ncgphysics.pdf

[3] M. Khalkhali. Basic Noncommutative Geometry. European Mathematical Society, 2009.

[4] J.C. Várilly. An Introduction to Noncommutative Geometry European Mathematical Society, 2006. https://arxiv.org/abs/physics/9709045

[5] M. Eckstein and B. Iochum. Spectral Action in Noncommutative Geometry. SpringerBriefs in Mathematical Physics. Springer, 2018, https://arxiv.org/abs/1902.05306.