Modeling Analysis and Control of Dynamical Systems Arising in Evolutionary Flow-Structure Systems with an Interface

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The Lecture is devoted to a presentation of PDE-models which describe complex dynamical systems occurring in modern applications which involve fluid and flow-structure interactions. These are coupled PDE-systems where coupling occurs on interface that separates two physical domains on which two different dynamical environments evolve (e.g. solid and fluid; or solid and wave). It is precisely the influence of the interface that plays a predominant role in determining the resulting dynamical properties of the overall coupled system. Its impact is reflected in the underlying technical analysis of the entire dynamical unit. The interface is the region where properties of a single dynamical component of one medium propagate onto the other medium, possibly changing drastically the properties of the second dynamical environment. Sometimes these changes are deleterious, even catastrophic; sometimes they are sought-after targets, very beneficial and desirable for the overall coupled system.

Illustrative examples include:
(i) elastic properties of an artery that effect the blood-flow within it, thus causing high blood pressure;
(ii) wind-induced vibrations of an oscillating structure that eventually determine fatigue failure (as in the collapse of the Tacoma Narrows Bridge);
(iii) dangerous vibrations of an airfoil due to strong headwinds, which may lead to loss of stability in flight; etc. Such examples are ubiquitous. For these, one would like, first, to determine the qualitative behavior of the uncontrolled solutions of the PDE-coupled system and, next, design suitable controllers - most desirably in feedback form and finite dimensional - capable to suppress or prevent instability and/or catastrophic regimes. The phenomenon of 'bad-outcome suppression' by feedback controllers is what is generally referred to as 'feedback stabilization' problem. Canonical illustrations may be: asymptotic noise suppression in an acoustic chamber or aircraft cockpit or cabin; asymptotic turbulence suppression of a fluid, etc.

Mathematical models describing the phenomena involve systems comprised of Euler equation linearised around unstable profile coupled on an interface with nonlinear system of dynamic elasticity. In recent years major advances have been made in the mathematical study of fluid-structure and flow-structure interactions. As a result, we now have an arsenal of mathematical tools and reasonably good understanding of issues related to well-posedness of these dynamics—both local and global, as well as their long time behavior and culminating with a finite dimensional description of asymptotic dynamics. The latter being amenable to finite dimensional control. One of the fundamental challenges is obtaining a good understanding of the propagation of either energy decay, or of the regularity properties, from one component of the coupled system to the other via interaction at the interface. Since the interface involves boundary traces of the respective PDE solutions, new developments in (hidden/sharp) regularity of traces play a key role.

Methods of (i) compensated compactness, (ii) microlocal analysis and (iii) hyperbolic dynamical system theory will be presented within the context of the described applications. Compensated compactness will be used in order to handle the nonlinear terms which are at the critical [with respect to Sobolev's embeddings] level. Microlocal analysis will be used in order to handle the linearization of the dynamics with boundary traces which are not controlled by the energy. Recently developed dynamical system theory for non-dissipative models will provide the main tool for obtaining the results on the existence of attracting sets and their properties.