

**The ultimate limits of privacy
and randomness...**

...for the paranoid ones



Artur Ekert

Outline

- **Is there a perfect cipher?**
- **Key distribution – the holy grail of cryptography**
- **Quantum physics comes to the rescue**
- **Less reality more security**

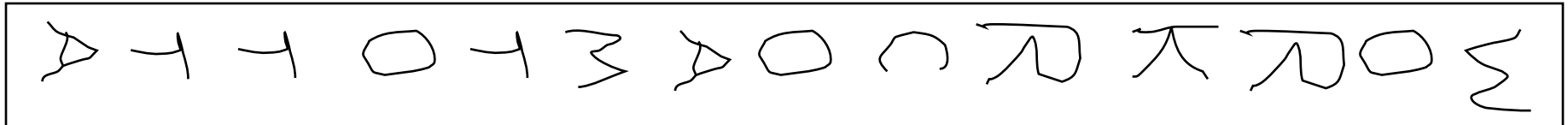
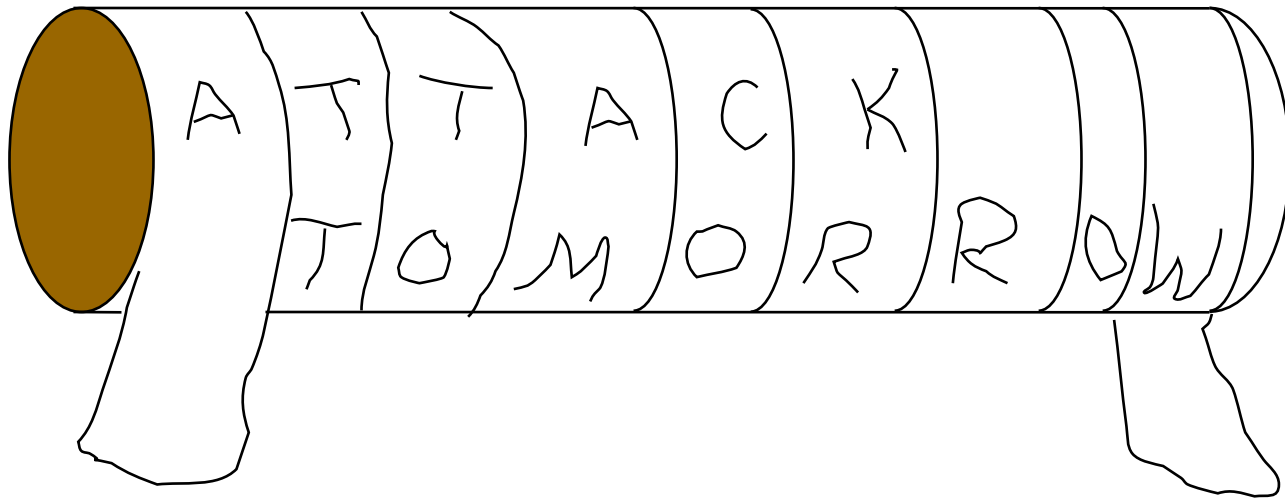
Basic techniques

- **PERMUTATIONS**
 - SCYTALE (400 BC)
- **SUBSTITUTIONS**
 - CAESAR SIPHER (50 BC)
- **PERMUTATIONS + SUBSTITUTIONS**

Scytale

400 BC

SPARTA



Permutation of characters

Caesar ciphers

50 BC

ROME

ABCDEFGHIJKLMNOPQRSTUVWXYZ
ABCDEFGHIJKLMNOPQRSTUVWXYZ

ABCDEFGHIJKLMNOPQRSTUVWXYZ
ABCDEFGHIJKLMNOPQRSTUVWXYZABC

A T T A C K T O M O R R O W
D W W D F N W R P R U U R Z

Code-makers versus code-breakers

**Julius Caesar
(100-44 BC)**



**Al Kindi
(800-873)**

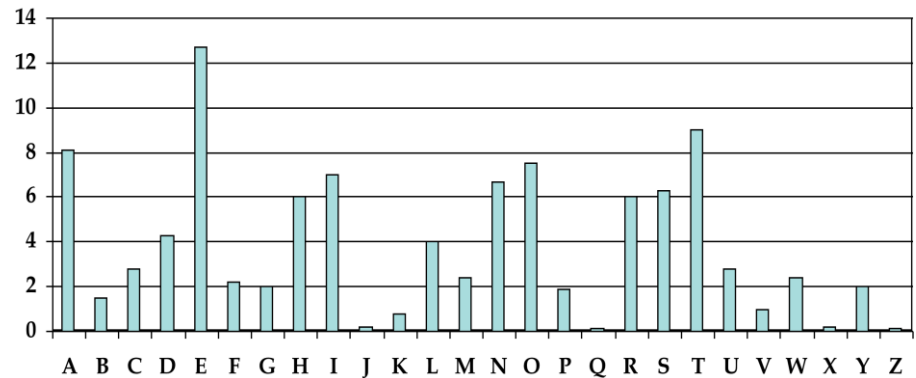


ABCDEFGHIJKLMNOPQRSTUVWXYZ

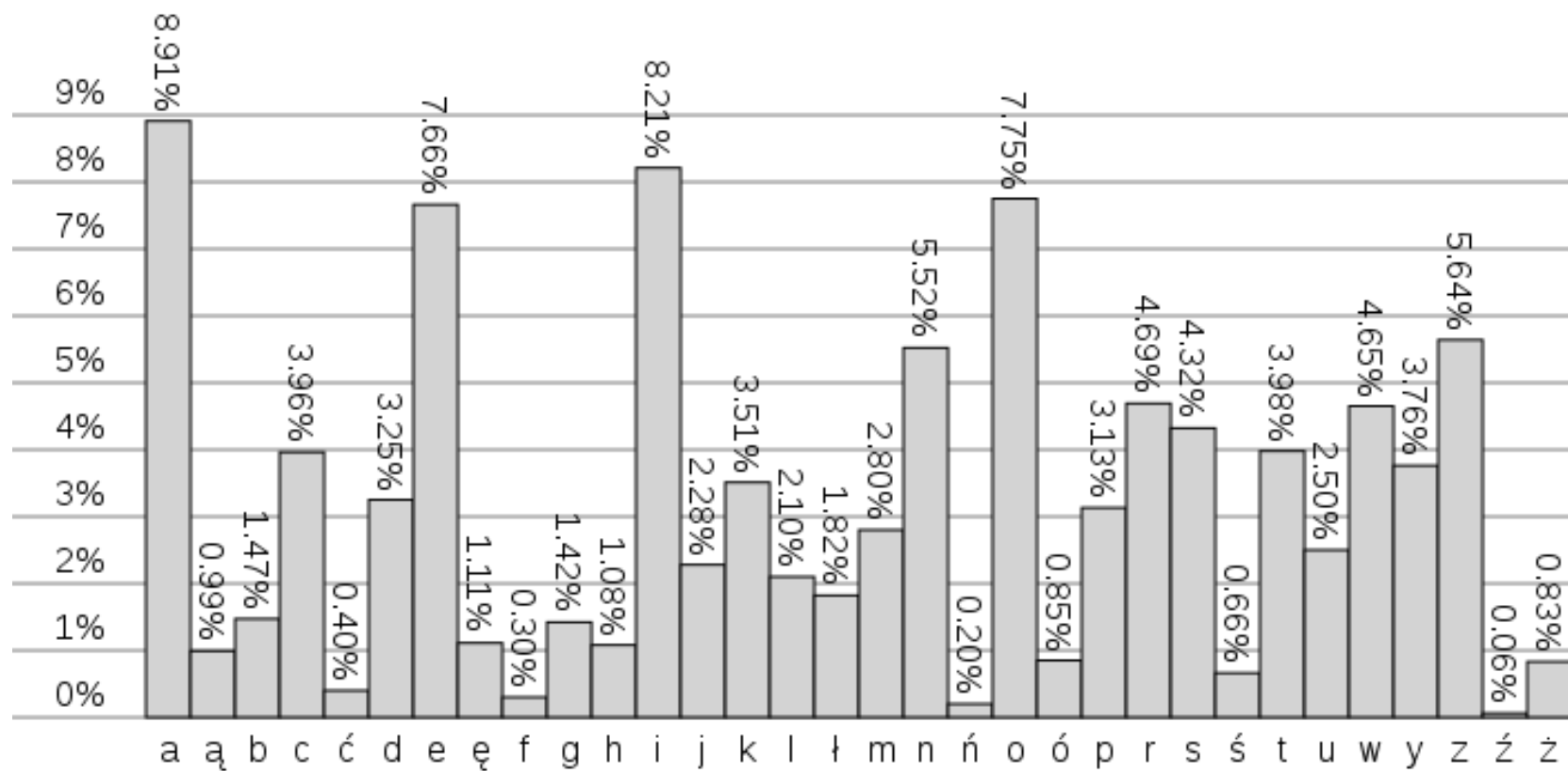


NWDEAPYFGTIJUKLMOZQRSBVCXH

$\approx 4 \times 10^{26}$ SUBSTITUTIONS



Frequency of letters in Polish



Counterexamples - Lipograms

That's right - this is a lipogram - a book, paragraph or similar thing in writing that fails to contain a symbol, particularly that symbol fifth in rank out of 26 (amidst 'd' and 'f') and which stands for a vocalic sound such as that in 'kiwi'. I won't bring it up right now, to avoid spoiling it...

The most famous lipogram: Georges Perec, *La Disparition* (1969) 85000 words without the letter e:

Tout avait l'air normal, mais tout s'affirmait faux. Tout avait l'air normal, d'abord, puis surgissait l'inhumain, l'affolant. Il aurait voulu savoir où s'articulait l'association qui l'unissait au roman : sur son tapis, assaillant à tout instant son imagination, ...

English translator, Gilbert Adair, in *A Void*, succeeded in avoiding the letter e as well

Gottlob Burmann (1737-1805) R-LESS POETRY. An obsessive dislike for the letter r; wrote 130 poems without using that letter, he also omitted the letter r from his daily conversation for 17 years...

Lipograms in Polish

Najbardziej znany polski lipogram został stworzony przez Juliana Tuwima i zamieszczony w tomie "Pegaz dęba". W utworze tym ani razu nie pojawia się litera "r", co widać w przytoczonym fragmencie:

"Słońce tego dnia wstało jakieś dziwnie leniwe,
matowe, bez blasku. Około południa na
powleczone niezwykłą bladością niebo wypełzły
zwały skłębionych żółtych obłoków i w jednej
chwili świat zasnuł się ciemnością".



Polyalphabetic ciphers

CODEMAKERS



Leone Battista Alberti
(1404-1472)

Johannes Trithemius
(1462-1516)

Blaise de Vigenere
(1523-1596)



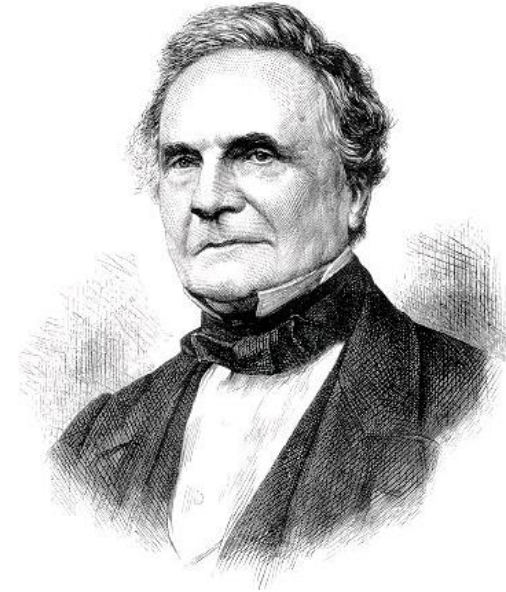
Alberti's encryption disk

Sequence of substitutions e.g.
7, 14, 19

Plaintext: S E L L

Cryptogram: Z S E S

CODEBREAKERS



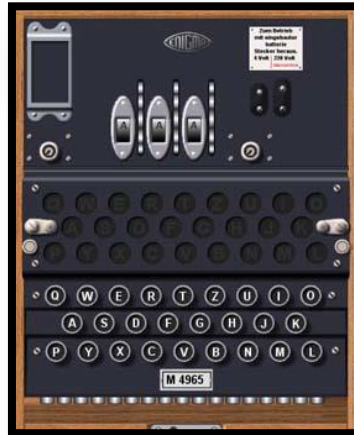
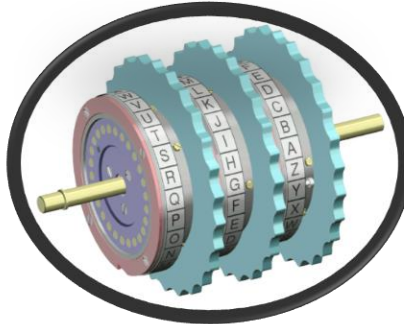
Charles Babbage
(1791-1871)

From Alberti's disk to rotor machines

CODEMAKERS



**Arthur Scherbius
(1878-1929)**



CODEBREAKERS



**Marian Rejewski
(1905-1980)**

The Poles who broke Enigma

(BS-4 Section)



Henryk Zygański.

Jerzy Różycki

Marian Rejewski



Maksymilian Ciężki



Gwido Langer

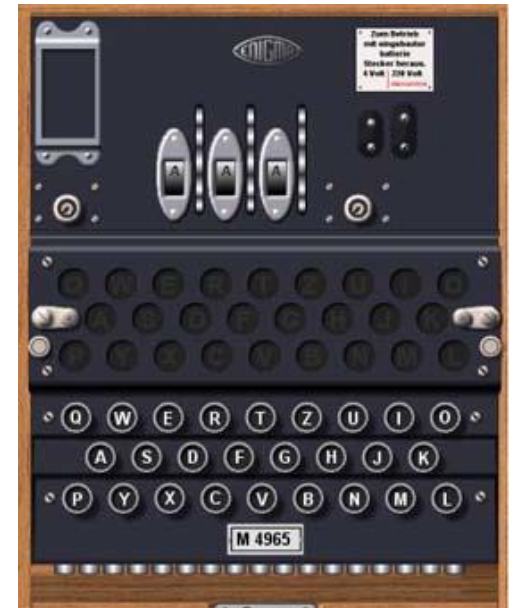
Is there a perfect cipher ?



SCYTALE 400BC



ALBERTI'S DISC 1450



ENIGMA 1940

One-time pad

message

0	1	1	1	0	1	0	0	1	1
---	---	---	---	---	---	---	---	---	---

key

0	1	0	1	1	1	0	0	1	0
---	---	---	---	---	---	---	---	---	---

cryptogram

0	0	1	0	1	0	0	0	0	1
---	---	---	---	---	---	---	---	---	---



0	0	1	0	1	0	0	0	0	1
---	---	---	---	---	---	---	---	---	---



0	0	1	0	1	0	0	0	0	1
---	---	---	---	---	---	---	---	---	---



0	0	1	0	1	0	0	0	0	1
0	1	0	1	1	1	0	0	1	0
0	1	1	1	0	1	0	0	1	1

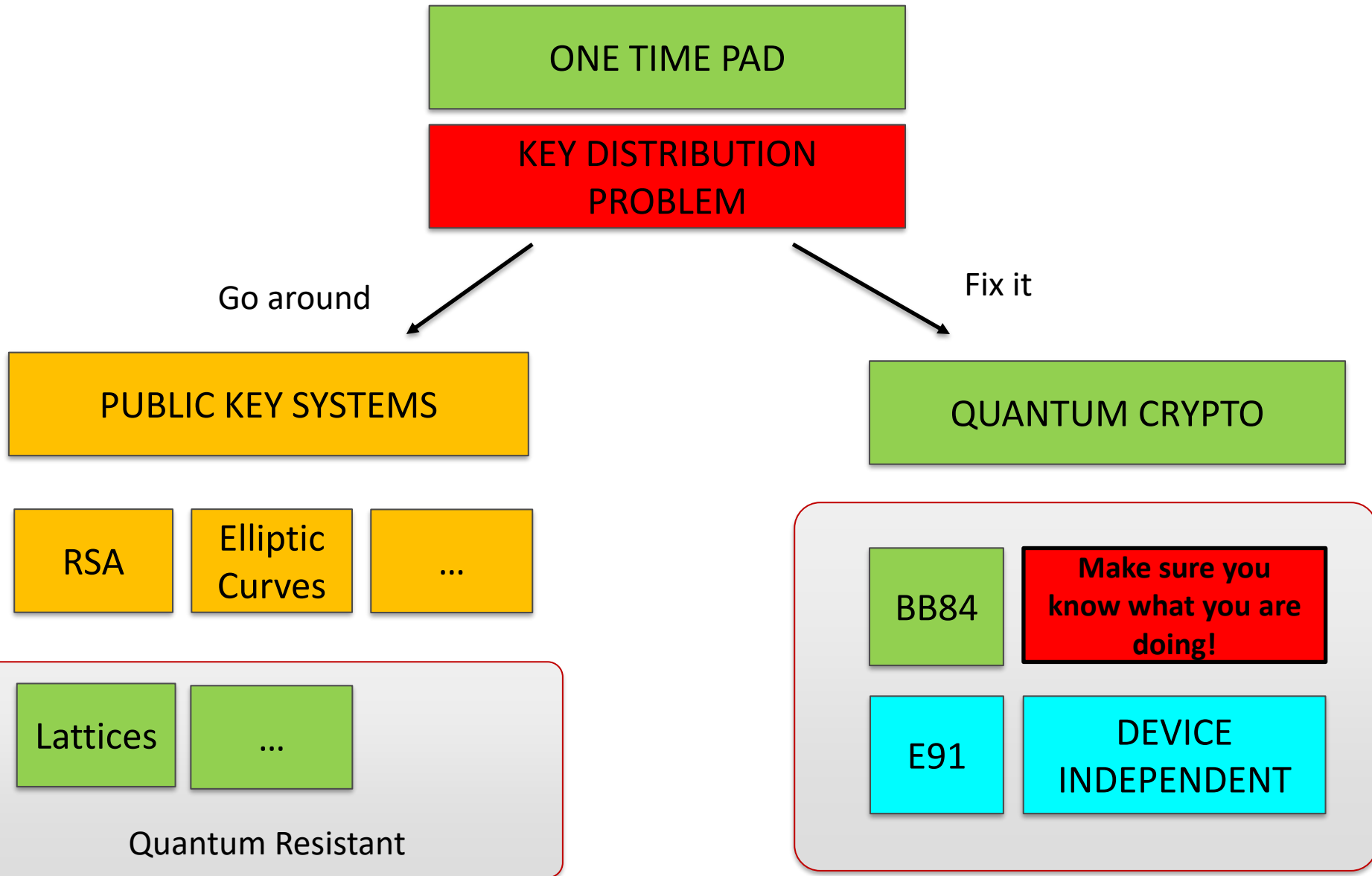
cryptogram

key

message

KEY DISTRIBUTION PROBLEM

Quest for perfect secrecy



Look it up - your homework 😊

- Public key cryptosystems: RSA, elliptic curves and lattice based

Post-quantum: there is still room for improvement

Report on the Security of LWE: Improved Dual Lattice Attack

The Center of Encryption and Information Security – MATZOV*†
IDF

Abstract

Many of the leading post-quantum key exchange and signature schemes rely on the conjectured hardness of the Learning With Errors (LWE) and Learning With Rounding (LWR) problems and their algebraic variants, including 3 of the 6 finalists in NIST's PQC process. The best known cryptanalysis techniques against these problems are primal and dual lattice attacks, where dual attacks are generally considered less practical.

In this report, we present several algorithmic improvements to the dual lattice attack, which allow it to exceed the efficiency of primal attacks. In the improved attack, we enumerate over more coordinates of the secret and use an improved distinguisher based on FFT. In addition, we incorporate improvements to the estimates of the cost of performing a lattice sieve in the RAM model, reducing the gate count of random product

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SOLILOQUY: A CAUTIONARY TALE

PETER CAMPBELL, MICHAEL GROVES AND DAN SHEPHERD

CESG, Cheltenham, UK

1. INTRODUCTION

The SOLILOQUY primitive, first proposed by the third author in 2007, based on cyclic lattices. It has very good efficiency properties, both in terms of public key size and the speed of encryption and decryption. There are straightforward techniques for turning SOLILOQUY into a key exchange or other public-key protocols. Despite these properties, we abandoned our search on SOLILOQUY after developing (2010 to 2013) a reasonably efficient quantum attack on the primitive. A similar quantum algorithm has been



Cryptology ePrint Archive

Paper 2022/214

Breaking Rainbow Takes a Weekend on a Laptop

Ward Beullens IBM Research – Zurich

Abstract

This work introduces new key recovery attacks against the Rainbow signature scheme, which is one of the three finalist signature schemes still in the NIST Post-Quantum Cryptography standardization project. The new attacks outperform previously known attacks for all the parameter sets submitted to NIST and make a key-recovery practical for the SL 1 parameters. Concretely, given a Rainbow public key for the SL 1 parameters of the second-round submission, our attack returns the corresponding secret key after on average 53 hours (one weekend) of computation time on a standard laptop.



Cryptology ePrint Archive

Paper 2022/975

An efficient key recovery attack on SIDH (preliminary version)

Wouter Castryck, KU Leuven

Thomas Decru, KU Leuven

Abstract

We present an efficient key recovery attack on the Supersingular Isogeny Diffie-Hellman protocol (SIDH), based on a "glue-and-split" theorem due to Kani. Our attack exploits the existence of a small non-scalar endomorphism on the starting curve, and it also relies on the auxiliary torsion point information that Alice and Bob share during the protocol. Our Magma implementation breaks the instantiation SIKEp434, which aims at security level 1 of the Post-Quantum Cryptography standardization process currently ran by NIST, in about one hour on a single core. This is a preliminary version of a longer article in preparation.

Key distribution problem



The key should be random, sufficiently long and secret (known only to Alice and Bob)



0	1	0	1	1	1	0	0	1	0
---	---	---	---	---	---	---	---	---	---

X



0	1	0	1	1	1	0	0	1	0
---	---	---	---	---	---	---	---	---	---

X

0	?	?	1	?	0	0	?	?	?
---	---	---	---	---	---	---	---	---	---

E

Probability of Eve guessing the key correctly should be very close to $\frac{1}{2^n}$

Privacy amplification



0	1	0	1	1	1	0	0	1	0
---	---	---	---	---	---	---	---	---	---

Alice and Bob can turn their partially secure key into a secure key as long as they can estimate how much Eve knows about the raw key.



0	1	0	1	1	1	0	0	1	0
---	---	---	---	---	---	---	---	---	---



BASIC IDEA

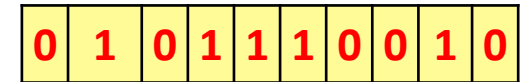
Suppose Eve knows one of the two bits, but Alice and Bob are not sure which one

$$X_1 X_2 \leftrightarrow Z = X_1 \oplus X_2$$

Privacy amplification



Alice and Bob can turn their partially secure key into a secure key as long as they can estimate how much Eve knows about the raw key.

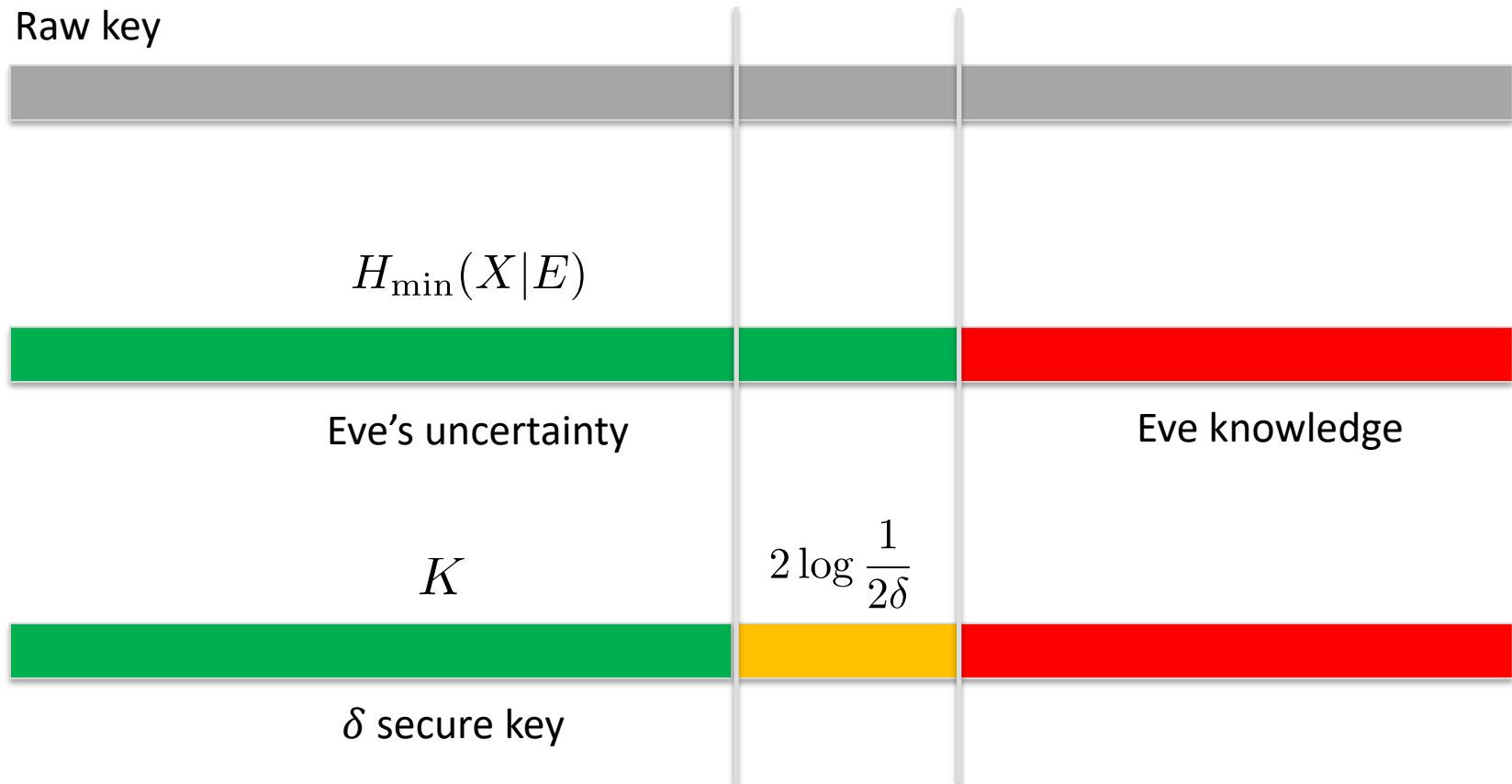


Probability of Eve guessing the key correctly should be very close to $\frac{1}{2^n}$

$$H_{\min}(X|E) = -\log p_{\text{guess}}(X|E)$$

$$l = H_{\min}(X|E) - 2 \log \frac{1}{2\delta}$$

The Leftover Hash Lemma



$$l = H_{\min}(X|E) - 2 \log \frac{1}{2\delta}$$

Look it up - your homework 😊

- Public key cryptosystems: RSA, elliptic curves and lattice based
- Randomness extractors and privacy amplification
- Why cryptographers use min-entropy rather than Shannon entropy?
- Define security using Kolmogorov / trace distance between probability distributions

How to find out how much Eve knows?



0	1	0	1	1	1	0	0	1	0
---	---	---	---	---	---	---	---	---	---

X

$$H_{\min}(X|E)$$



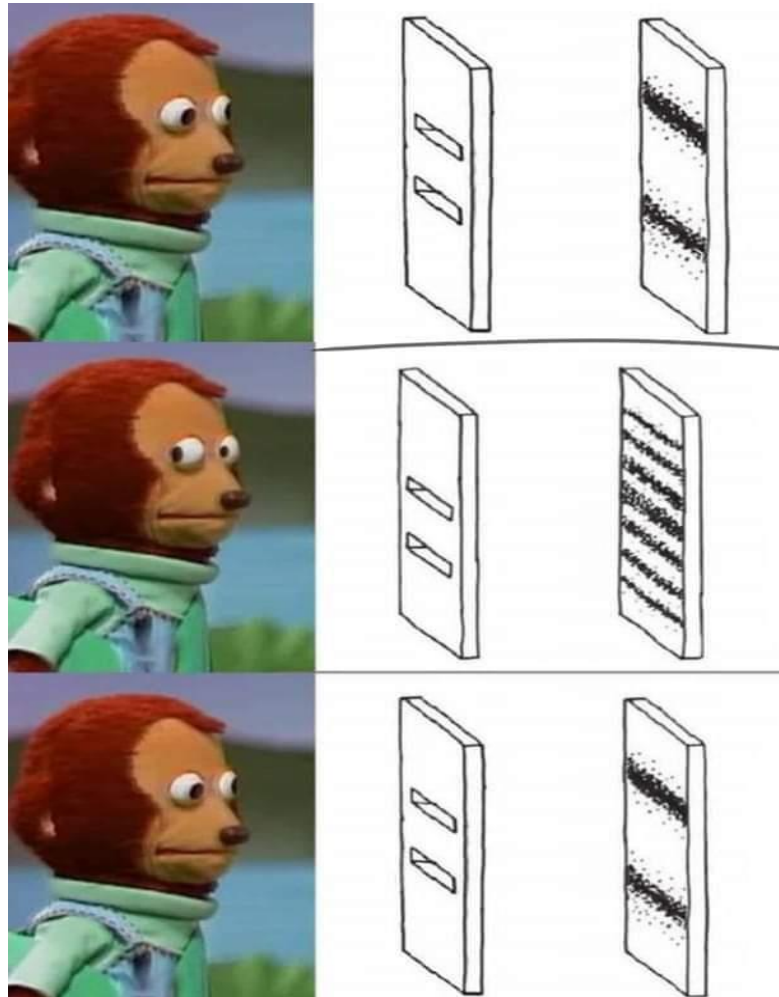
0	1	0	1	1	1	0	0	1	0
---	---	---	---	---	---	---	---	---	---

X

0	?	?	1	?	0	0	?	?	?
---	---	---	---	---	---	---	---	---	---

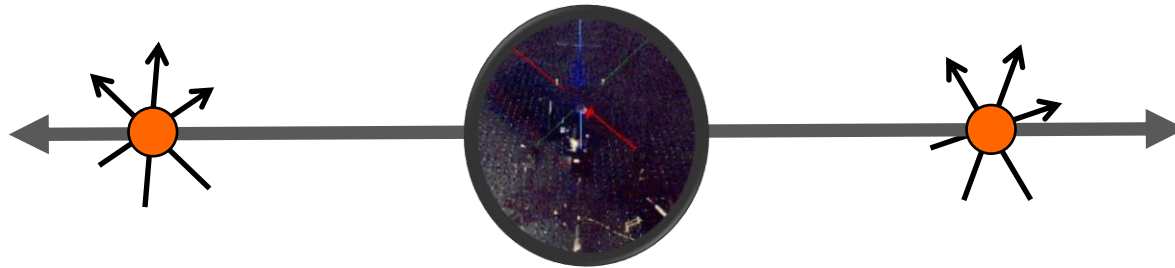
E

Why quantum in cryptography?

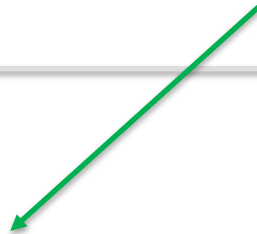


“Watching” does make a difference

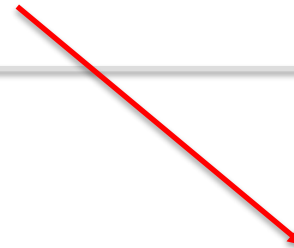
Use entanglement!



$$\frac{1}{\sqrt{2}}(|0\rangle|0\rangle + |1\rangle|1\rangle)$$



$$\frac{1}{\sqrt{2}}(|0\rangle|0\rangle + |1\rangle|1\rangle) \otimes |e\rangle$$



$$\frac{1}{\sqrt{2}}(|0\rangle|0\rangle \otimes |e_0\rangle + |1\rangle|1\rangle \otimes |e_1\rangle)$$

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- Quantum entanglement

Quantum cryptography

PHYSICAL REVIEW LETTERS

VOLUME 67

5 AUGUST 1991

NUMBER 6

Quantum Cryptography Based on Bell's Theorem

Artur K. Ekert

Merion College and Physics Department, Oxford OX1 3PU, United Kingdom
(Received 18 April 1991)

Practical application of the generalized Bell's theorem in the so-called key distribution process in cryptography is reported. The proposed scheme is based on the Bohm's version of the Einstein-Podolsky-Rosen Gedankenexperiment and Bell's theorem is used to test for eavesdropping.

QUANTUM CRYPTOGRAPHY: PUBLIC KEY DISTRIBUTION AND COIN TOSsing
Charles H. Bennett (IBM Research, Yorktown Heights NY 10598 USA)
Gilles Brassard (Dept. 180, Univ. de Montreal, H3C 3J7 Canada)

When elementary quantum systems, such as polarized photons, are used to transmit digital information, the uncertainty principle gives rise to novel cryptographic phenomena unachievable with traditional transmission media, e.g., a communication channel in which it is impossible to counterfeit, and multiplexing two or three messages in such a way that reading one destroys the others. More recently (1980), quantum coding has been used in conjunction with

Submitted to IEEE, Information Theory ca 1970. Later published in SIGACT News 15(1, 78-88 (1988))

This paper treats a class of codes made possible by restrictions on measurement related to the uncertainty principle. Two concrete examples and some general results are given.

Conjugate Coding

Stephen Wiesner

Columbia University, New York, N.Y.
Department of Physics

The uncertainty principle imposes restrictions on the capacity of certain types of communication channels. This paper will show that in compensation for this "quantum noise", quantum mechanics allows us novel forms of coding without analogue in communication channels adequately described by classical physics.

* Research supported in part by the National Science Foundation.

STEVEN WIESNER
1970



CHARLES H. BENNETT
GILLES BRASSARD
1984



ARTUR EKERT
1991



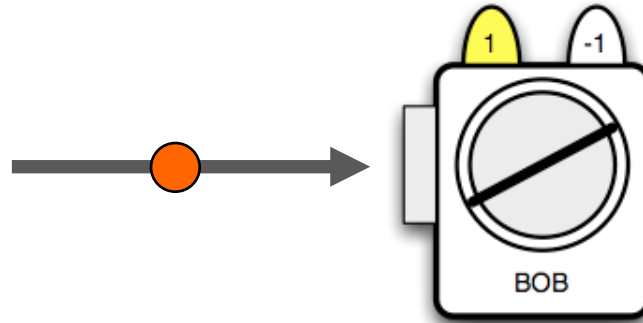
PREPARE &
MEASURE

ENTANGLEMENT
BASED

SECURITY PROOFS
EXPERIMENTS
PROTOTYPES
PRODUCTS

Device independence etc

Polarization



POLARIZATION IS AN INTRINSIC PROPERTY OF A PHOTON

WE CANNOT JUST “MEASURE POLARIZATION” - WE CAN ONLY MEASURE POLARIZATION WITH RESPECT TO SOME SPECIFIED DIRECTION

IN ANY MEASUREMENT WE CAN GET ONLY TWO RESULTS: +1 OR -1

The story of worry

MAY 15, 1935

PHYSICAL REVIEW

VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in

quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

1.

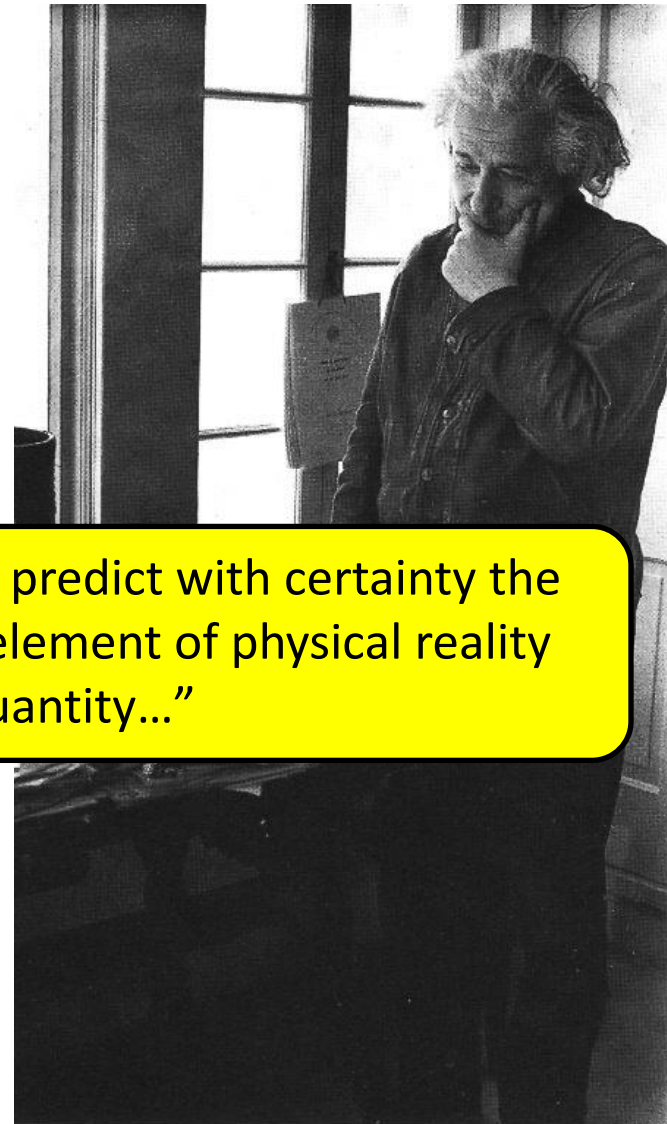
ANY serious consideration of a physical theory must take into account the distinction between the objective reality, which is independent of any theory, and the physical

Whatever the meaning assigned to the term *complete*, the following requirement for a complete theory seems to be a necessary one: *every element of the physical reality must have a counterpart in the physical theory*. We shall call this the

“...If without any way disturbing a system, we can predict with certainty the value of a physical quantity then there exists an element of physical reality corresponding to this physical quantity...”

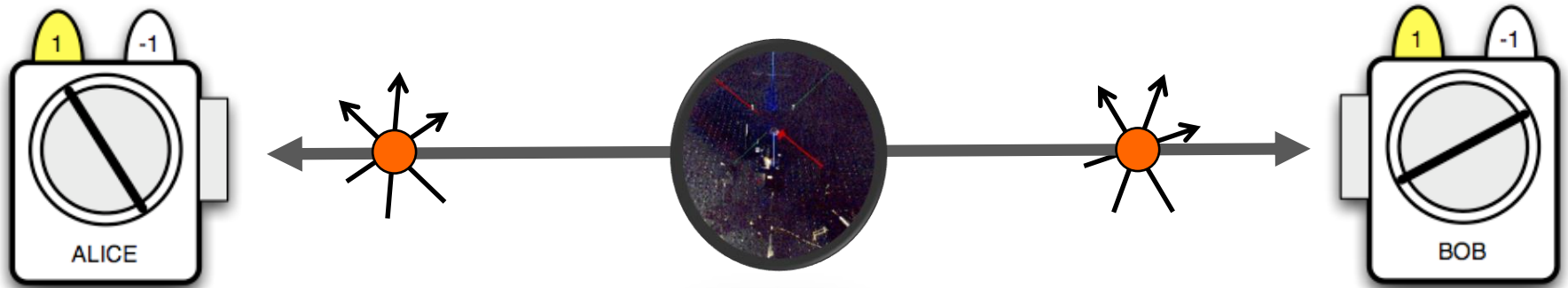
It is only in the case in which positive answers may be given to both of these questions, that the concepts of the theory may be said to be satisfactory. The correctness of the theory is judged by the degree of agreement between the conclusions of the theory and human experience. This experience, which alone enables us to make inferences about reality, in physics takes the form of experiment and measurement. It is the second question that we wish to consider here, as applied to quantum mechanics.

comprehensive definition of reality is, however, unnecessary for our purpose. We shall be satisfied with the following criterion, which we regard as reasonable. *If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.* It seems to us that this criterion, while far from exhausting all possible ways of recognizing a physical reality, at least provides us with one



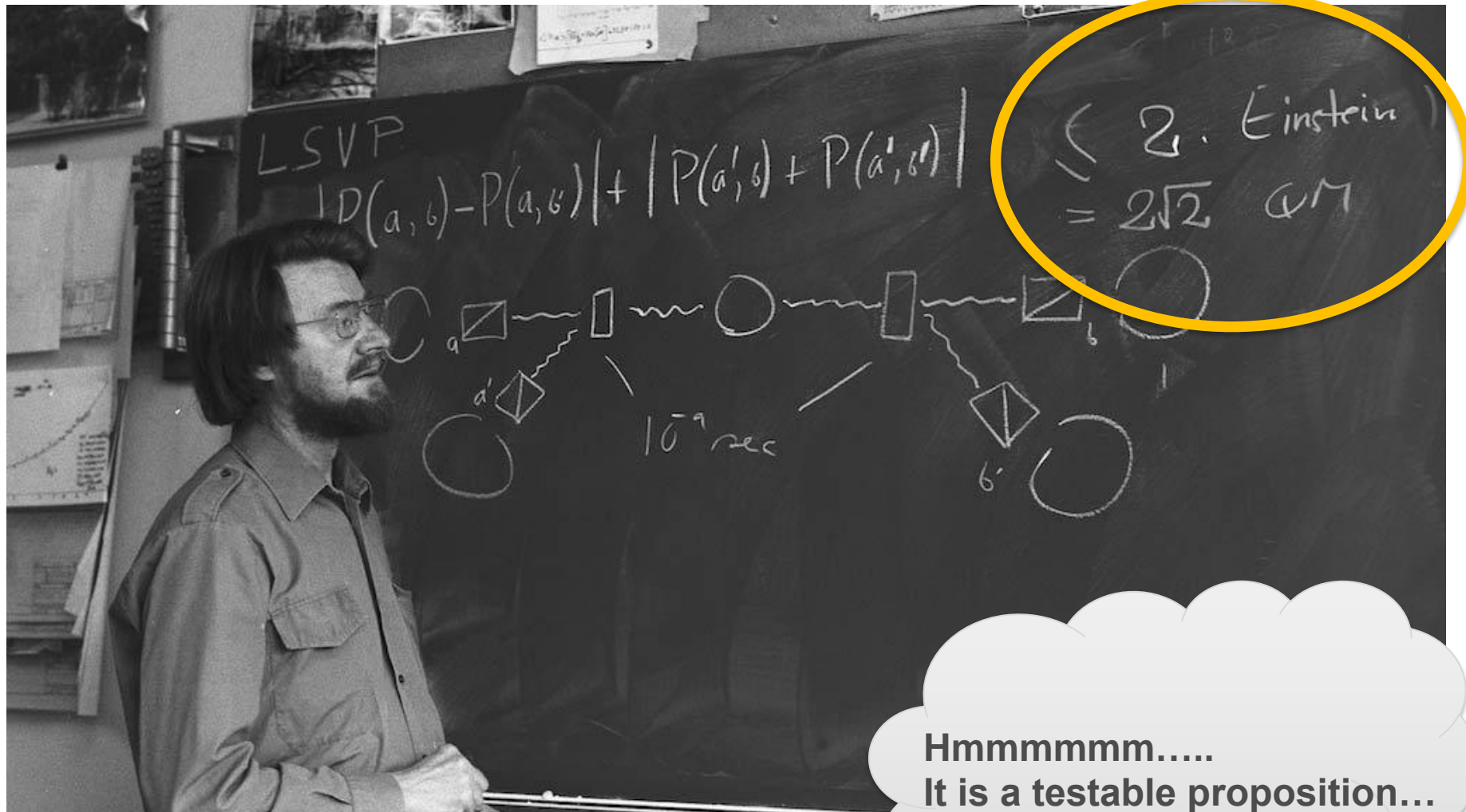
DEFINITION OF EAVESDROPPING

Predetermined or not?



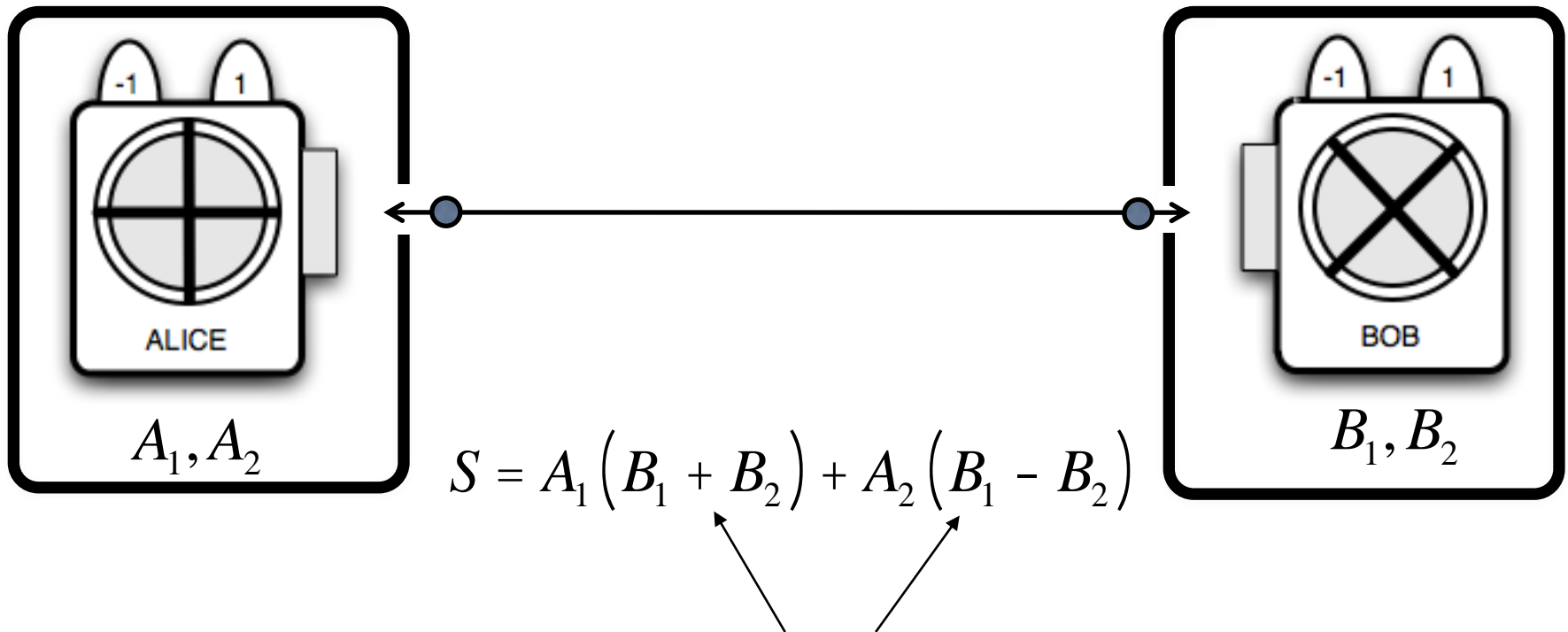
Do photons have predetermined values of polarizations?

Enter John Bell



year 1964

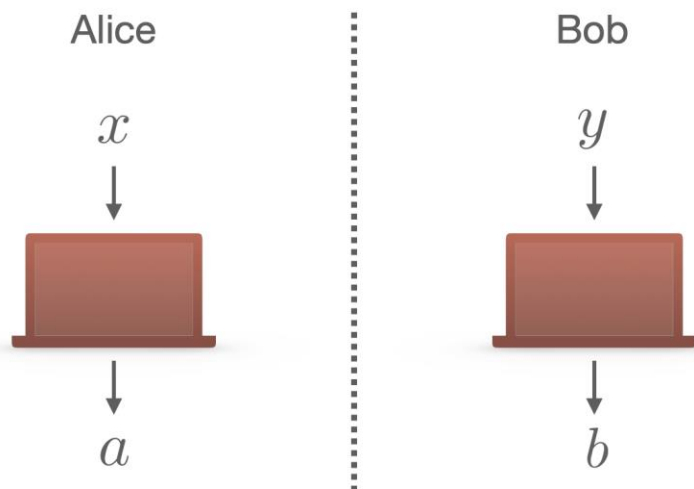
Bell's inequalities...



One of these terms is 0 and the other is ± 2

$$S = \pm 2 \quad \text{hence} \quad -2 \leq \langle S \rangle \leq 2$$

More recent take on Bell's inequalities



CHSH Game:

Alice:	Input	$x \in \{0, 1\}$
	Output	$a \in \{0, 1\}$
Bob:	Input	$y \in \{0, 1\}$
	Output	$b \in \{0, 1\}$
Win:	$a \oplus b = x \cdot y$	

- ▶ Best classical strategy: 75% winning probability $p(ab|xy) = \sum_{\lambda} p(\lambda)p(a|x\lambda)p(b|y\lambda)$
- ▶ Best quantum strategy: ~85% winning probability $|\Phi^+\rangle_{AB}$

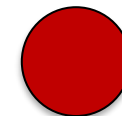
Shared randomness



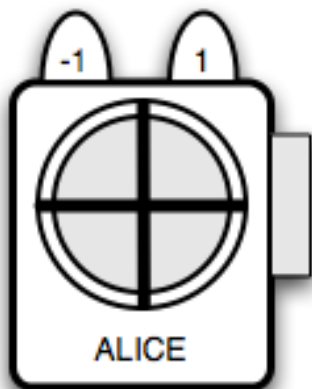
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- Define security using Kolmogorov / trace distance between probability distributions
- Quantum entanglement
- CHSH non-local game

Local realism can be refuted...

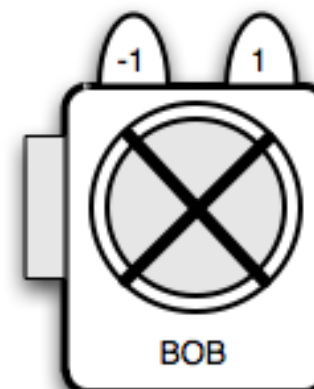


Experimental fact



A_1, A_2

If A and B are θ degrees apart Alice's and Bob's results agree with the probability $\sin^2 \frac{\theta}{2}$



B_1, B_2

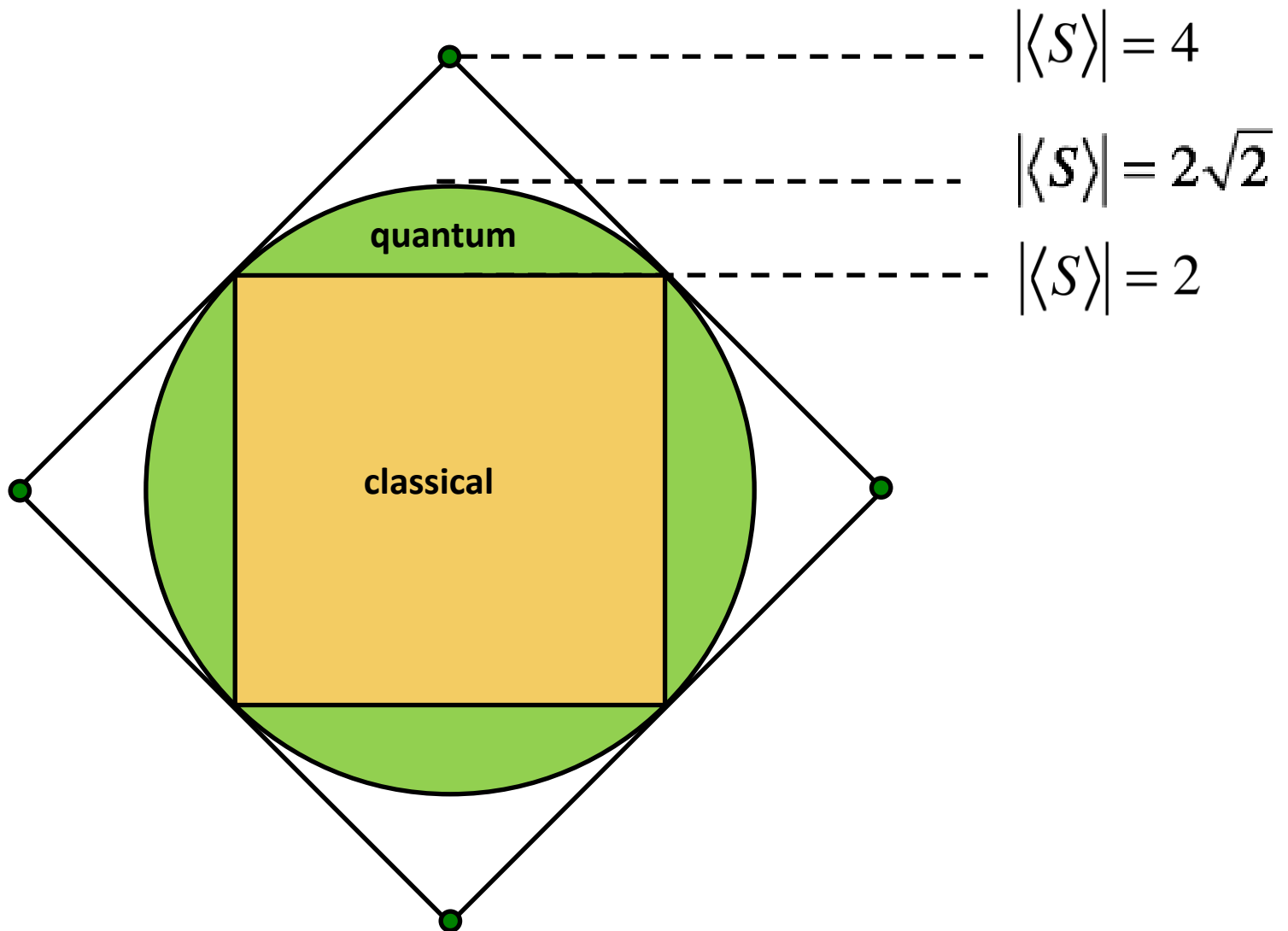
Results agree: $AB = 1$

Results disagree: $AB = -1$

$$\langle AB \rangle = \sin^2 \left(\frac{\theta}{2} \right) - \cos^2 \left(\frac{\theta}{2} \right) = -\cos \theta$$

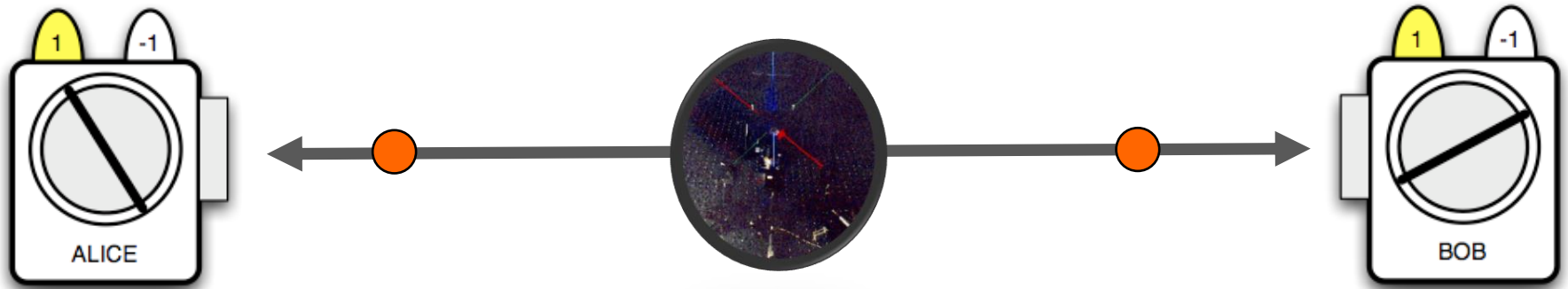
$$-2\sqrt{2} \leq \langle A_1 B_1 \rangle - \langle A_1 B_2 \rangle + \langle A_2 B_1 \rangle + \langle A_2 B_2 \rangle \leq 2\sqrt{2}$$

Correlations galore



Polytope of non-signaling correlations

Less reality more security



PHOTONS DO NOT CARRY PREDETERMINED VALUES OF POLARIZATIONS

IF THE VALUES DID NOT EXIST PRIOR TO MEASUREMENTS THEY WERE NOT AVAILABLE TO ANYBODY INCLUDING EAVESDROPPERS

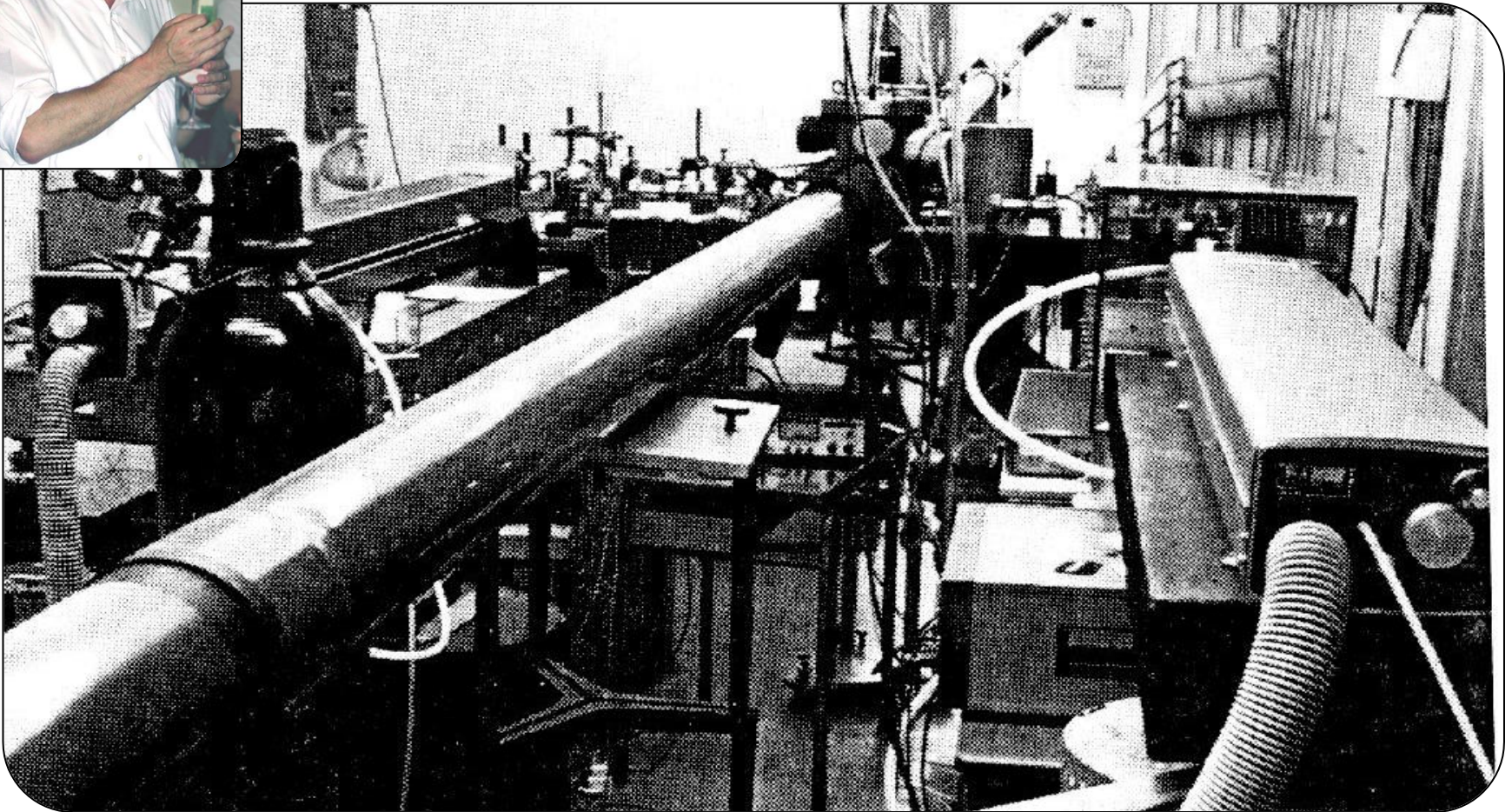
TESTING FOR THE VIOLATION OF BELL'S INEQUALITIES = TESTING FOR EAVESDROPPING

Alain Aspect and his quantum magic



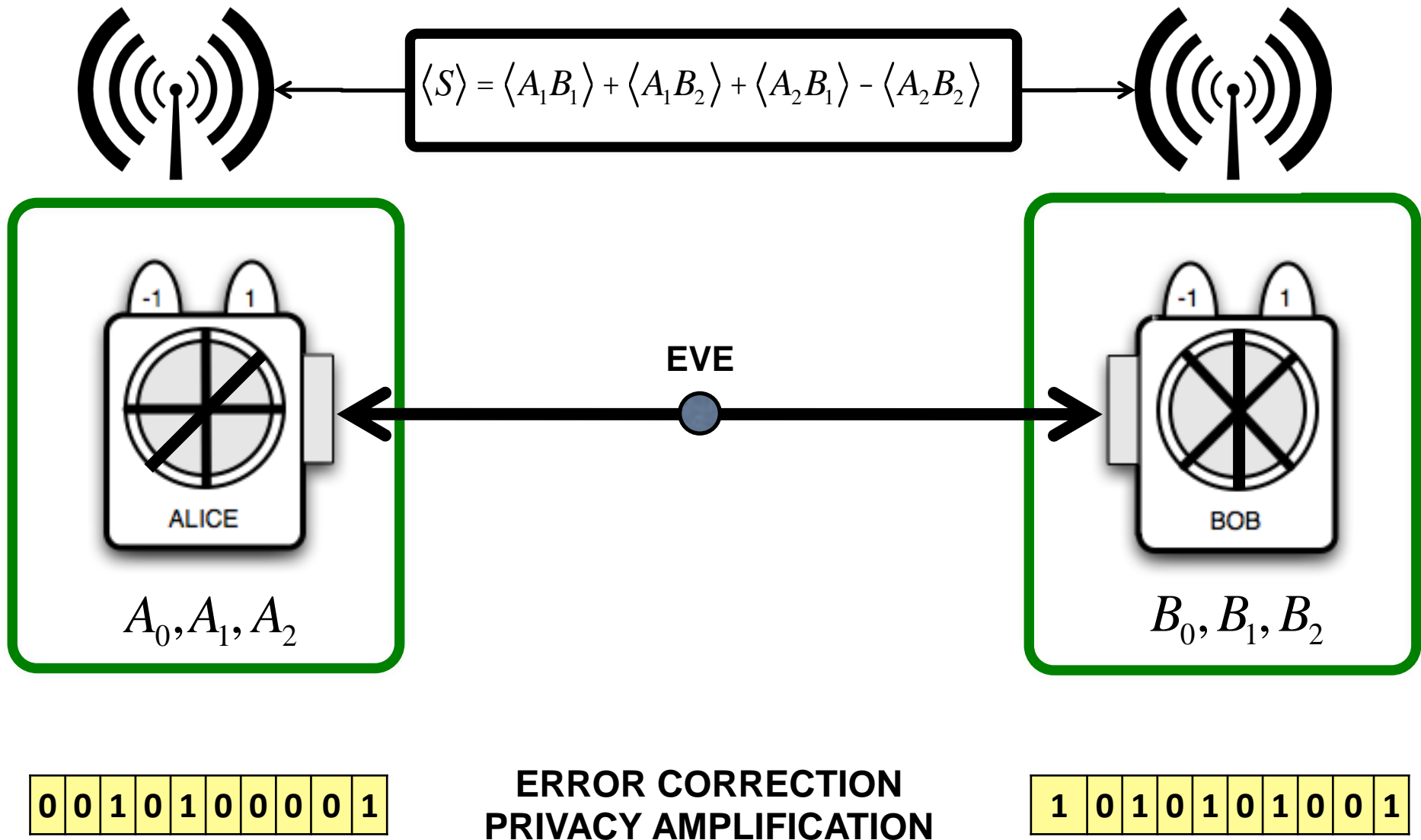
$$S > 2$$

Et voilà!



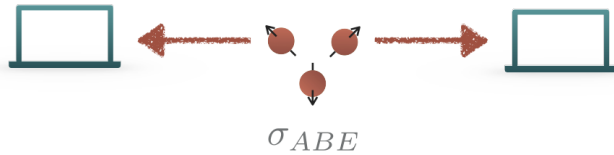
Institut d'Optique d'Orsay (1982)

Bell inequalities and security



You need some mathematical gymnastics

Eve uses the same strategy in each round, independently of all other rounds

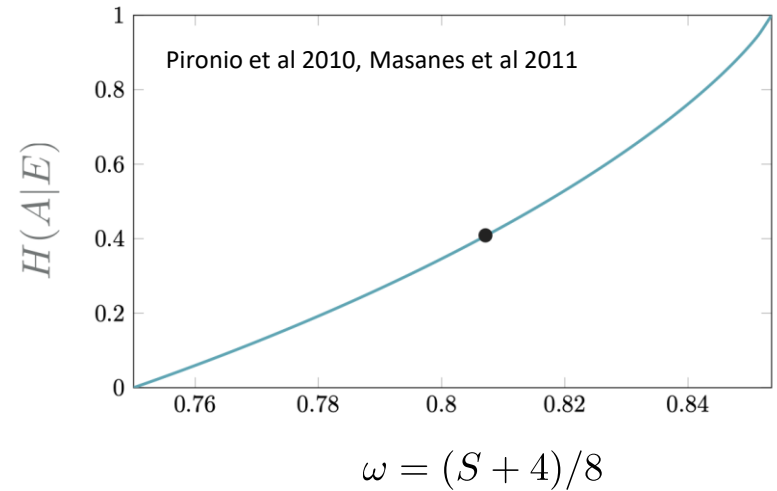


S

$$H_{\min}^{\varepsilon}(\mathbf{A}|\mathbf{E})_{\rho} \geq nH(A|E)_{\sigma} - c_{\varepsilon}\sqrt{n}$$

Extractors

Secret key



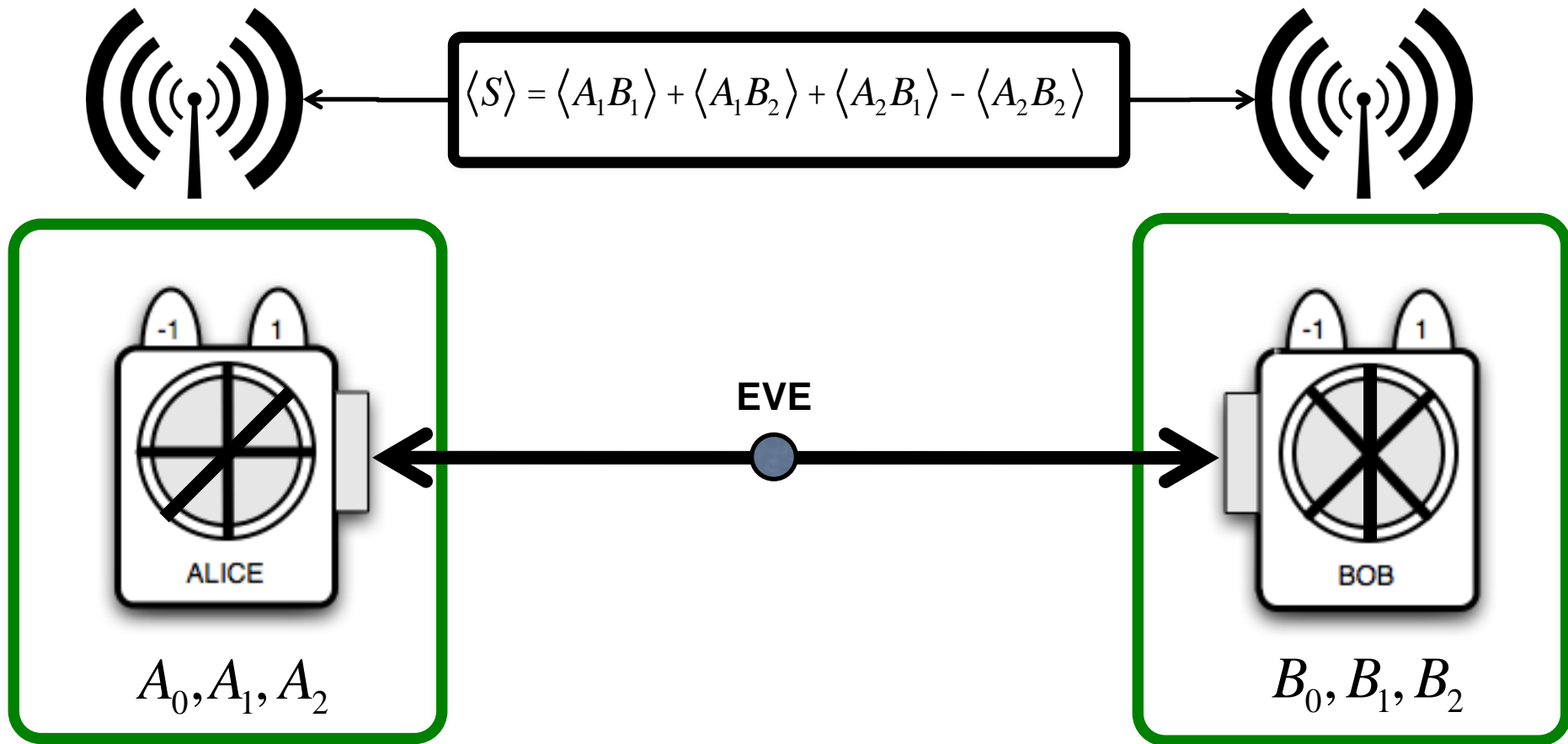
Quantum Asymptotic Equipartition Property
M. Tomamichel et al (2009) IDD CASE

Eve distributes the key!

Look it up - your homework 😊

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- CHSH non-local game
- Quantum Asymptotic Equipartition Property for entropy

Secure as long as...



- Alice's and Bob's labs are secure - no information leaks
- Alice and Bob control and trust devices in their labs
- Alice and Bob have free will and can choose their observables

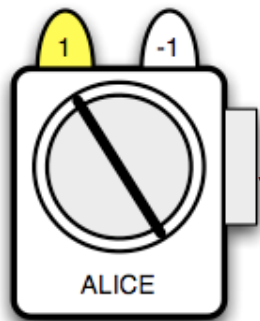
And all this can be demonstrated...

Parametric down conversion

Entangled photons



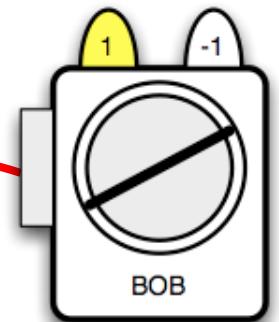
Optical fibers



Polarizing filters
& photodetectors

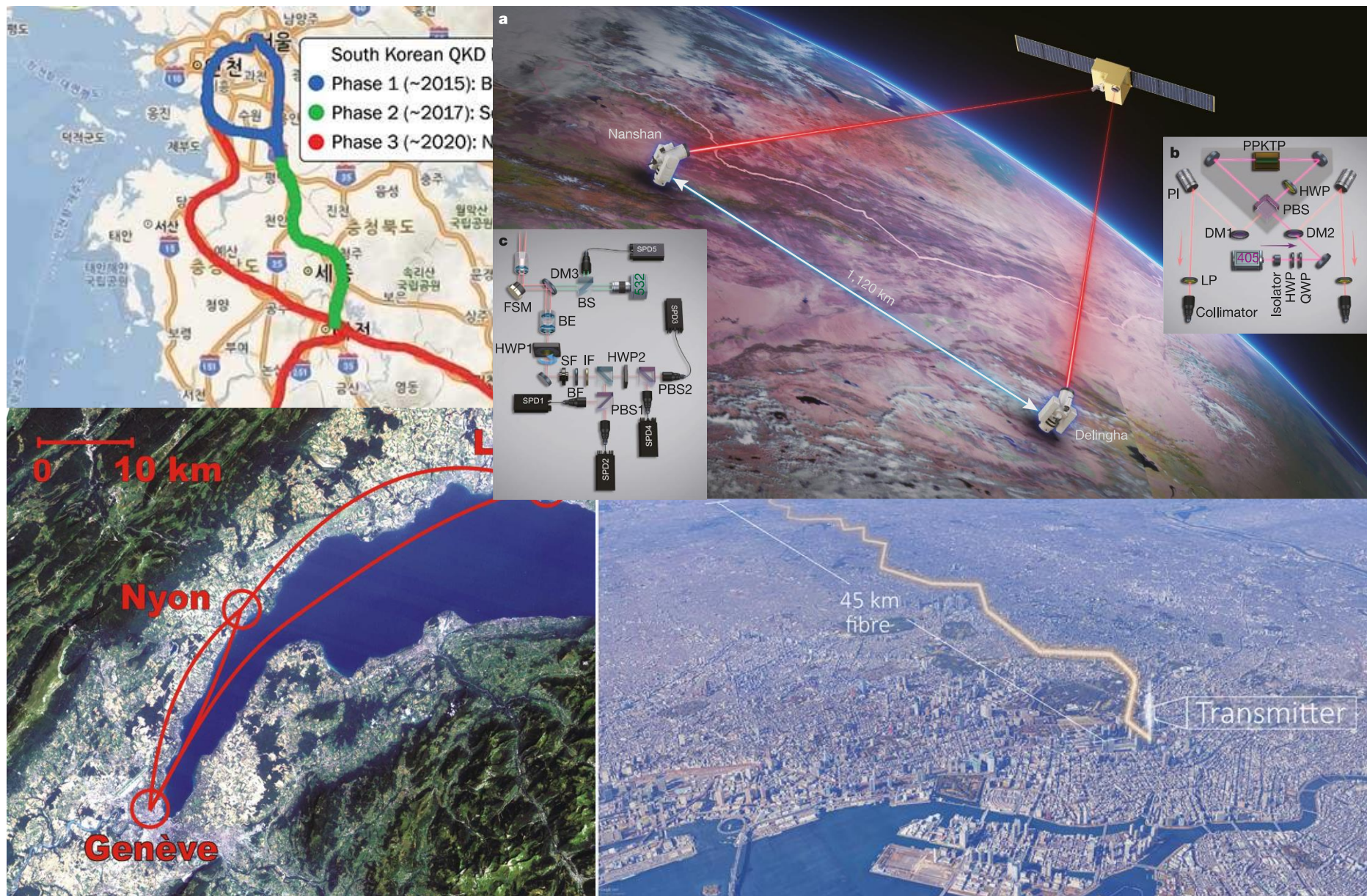


DRA MALVERN - OXFORD 1991



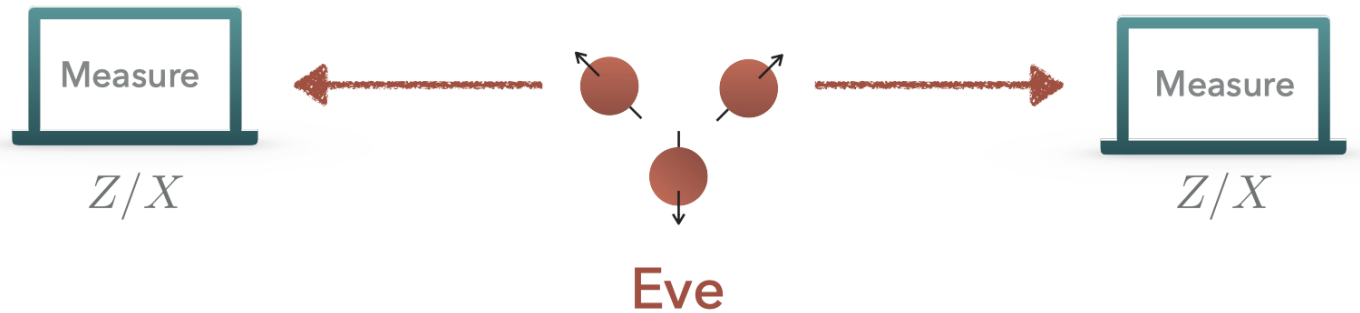
Polarizing filters
& photodetectors

...and implemented

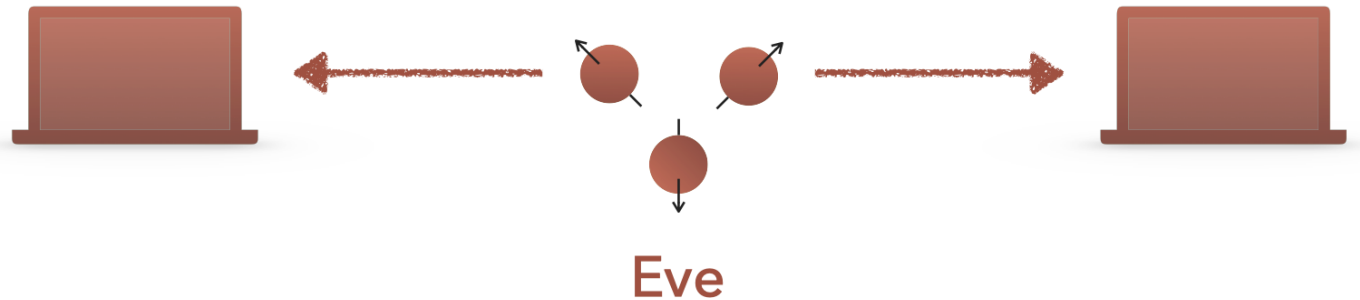


At the mercy of Eve

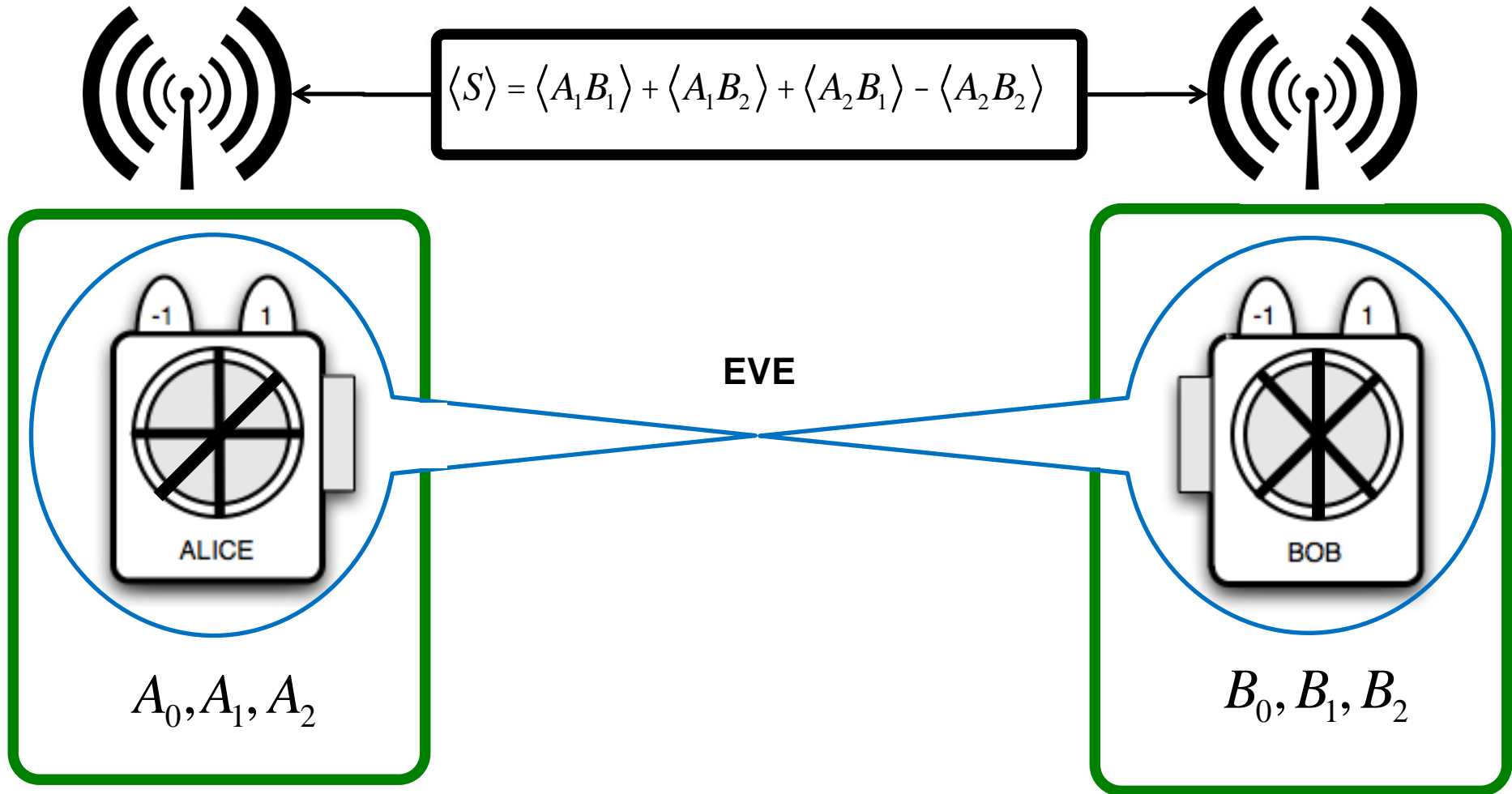
Ekert 91



Device-independent



Device independent



- Alice's and Bob's labs are secure - no information leaks
- Alice and Bob control and trust devices in their labs
- Alice and Bob have free will and can choose their observables

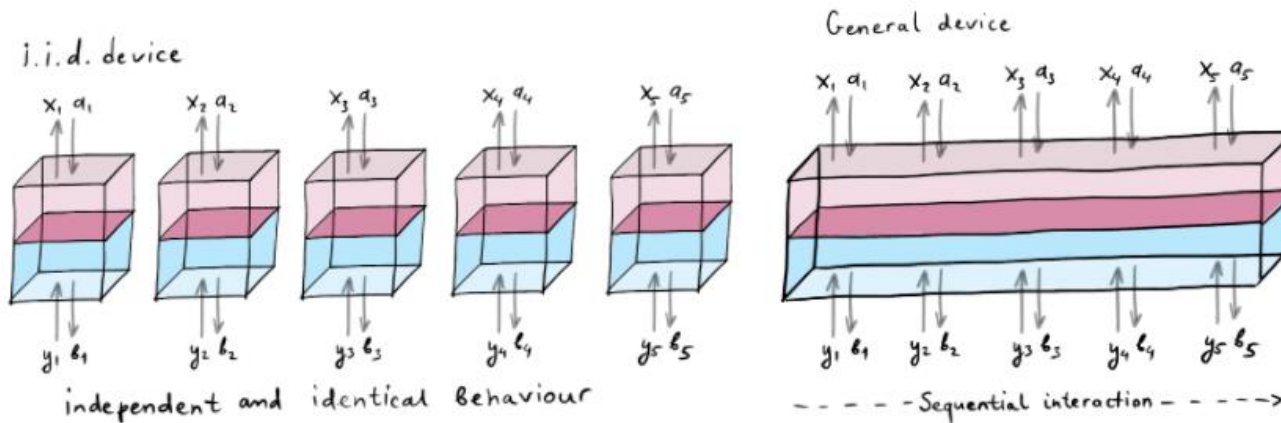
Towards device-independent crypto

A. Acin, N. Brunner, N. Gisin, S. Massar, V. Scarani



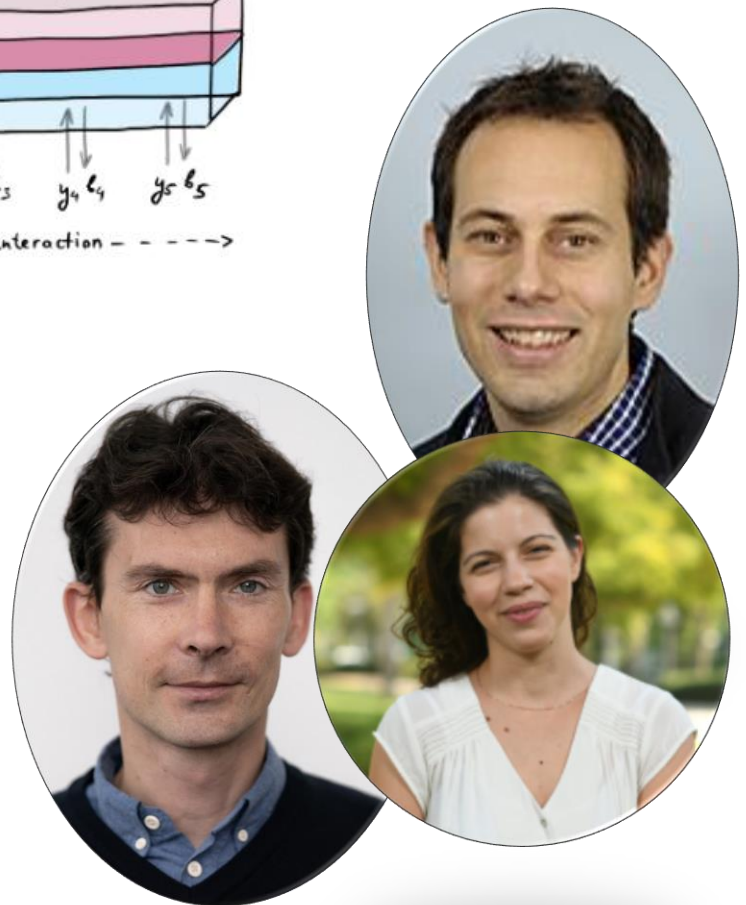
Courtesy Rotem Arnon-Friedman

EAT...



Entropy Accumulation Theorem (EAT) allows us to reduce arbitrary strategies to i.i.d. strategies and enables simple device-independent security proofs.

Rotem Arnon-Friedman, Renato Renner and Thomas Vidick.
Simple and tight device-independent security proofs.
SIAM J. Comput. **48**, 181 (2019). doi: [10.1137/18M1174726](https://doi.org/10.1137/18M1174726)



You can have your key and EAT it

1. Winning a non-local game

$$H(A|E) \geq f(\text{win prob.})$$



2. Entropy accumulation
(Reduction to IID)

$$H_{\min}^{\varepsilon}(\mathbf{A}|\mathbf{E})_{\rho} \geq nH(A|E)_{\sigma} - c_{\varepsilon}\sqrt{n}$$



3. Quantum-proof extractors

$$\|\rho_{\text{Ext}(A,S)SE} - \rho_{U_{\ell}} \otimes \rho_{SE}\| \leq \varepsilon$$



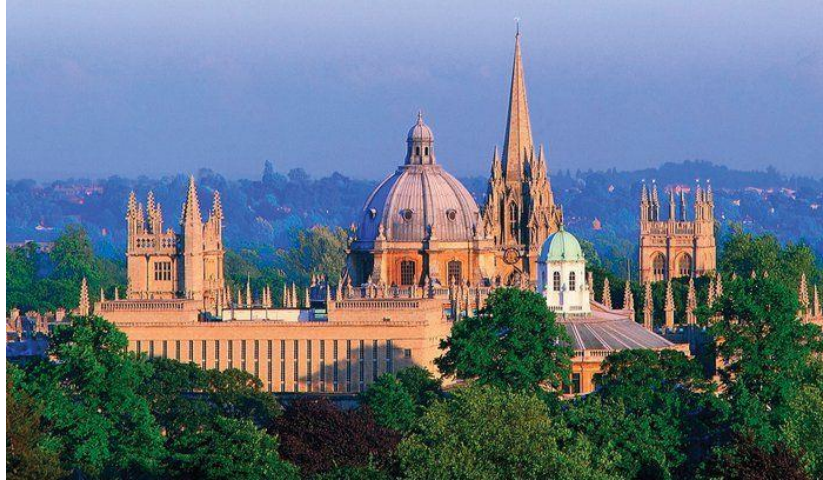
4. Secrecy

$$(1 - \Pr(\text{abort})) \|\rho_{K_A E} - \rho_{U_{\ell}} \otimes \rho_E\| \leq \varepsilon_{\text{sec}}$$

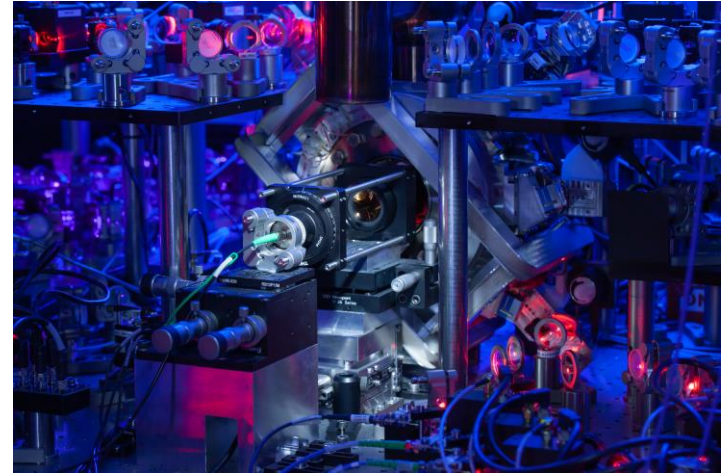
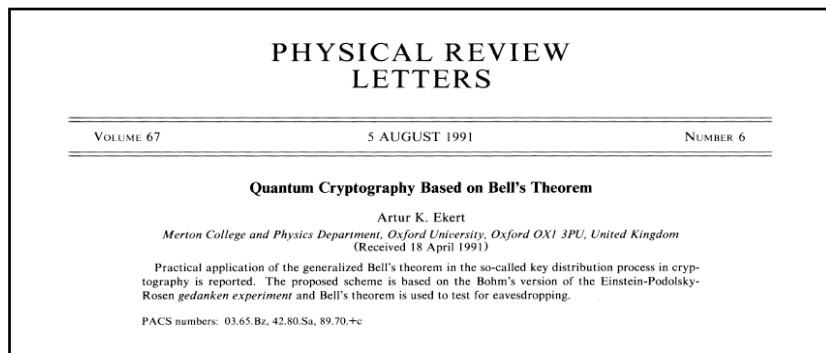
Look it up - your homework 😊

- Public key cryptosystems: RSA, elliptic curves and lattice based
- Randomness extractors and privacy amplification
- Why cryptographers use min-entropy rather than Shannon entropy?
- Define security using Kolmogorov / trace distance between probability distributions
- Quantum entanglement
- CHSH non-local game
- Quantum Asymptotic Equipartition Property for entropy
- Entropy Accumulation Theorem (EAT) – a real challenge 😊

It is not true that nothing changes in Oxford



From Oxford in 1991...



...to Oxford 2021

Article

Experimental quantum key distribution certified by Bell's theorem

<https://doi.org/10.1038/s41586-022-04941-5>

Received: 29 September 2021

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Published online: 27 July 2022

Check for updates

D. P. Nadlinger^{1,2,3}, P. Drmota¹, B. C. Nichol¹, G. Araneda¹, D. Main¹, R. Srinivas¹, D. M. Lucas¹, C. J. Ballance^{1,2,3}, K. Ivanov², E. Y.-Z. Tan³, P. Sekatski⁴, R. L. Urbanke², R. Renner², N. Sangouard^{4,5,6} & J.-D. Bancal^{4,5,6}

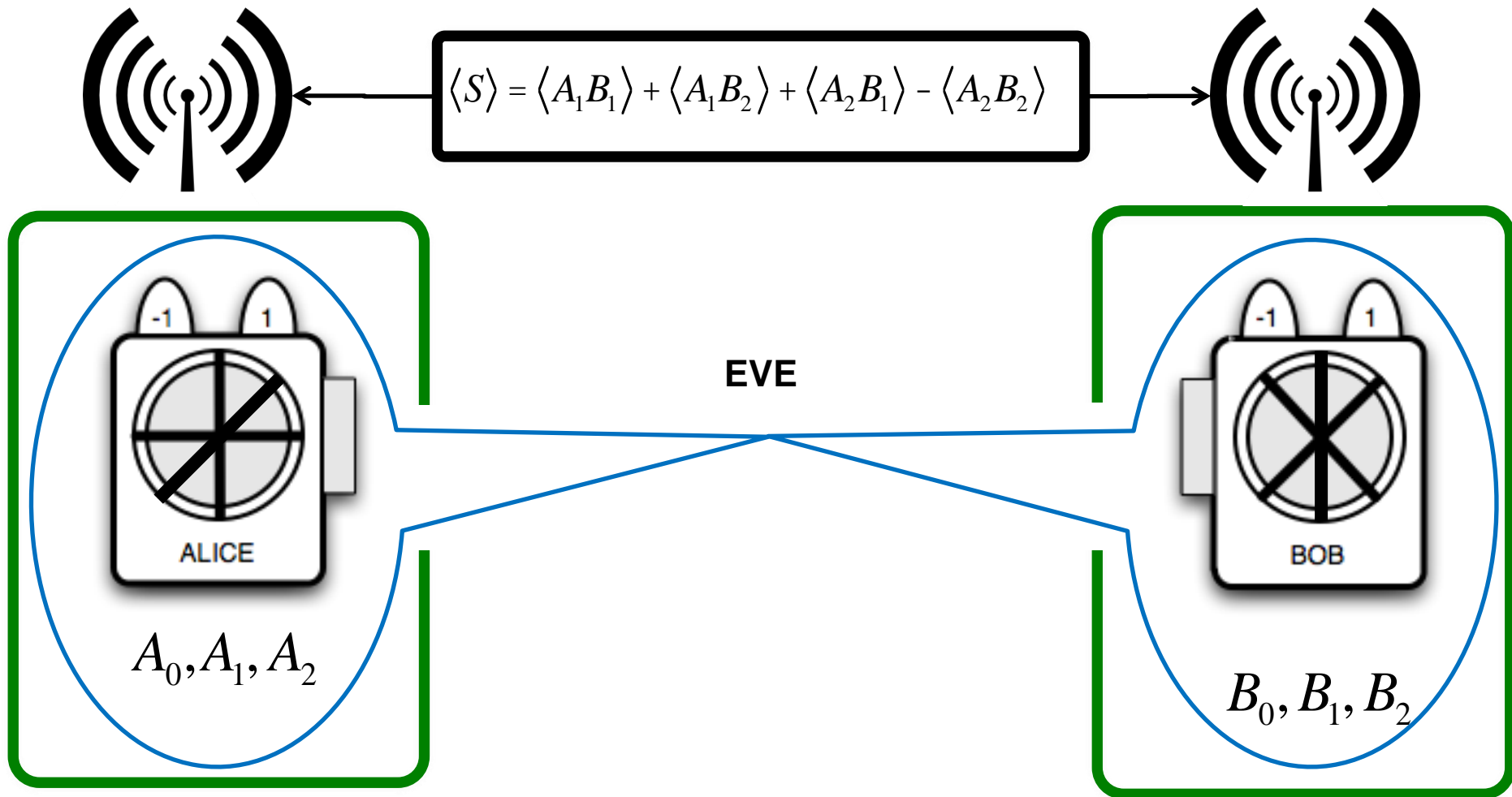
Cryptographic key exchange protocols traditionally rely on computational conjectures such as the hardness of prime factorization¹ to provide security against eavesdropping attacks. Remarkably, quantum key distribution protocols such as the

End of worries?



You need perfect randomness, right ?

Device independent & “partial free will”



- Alice's and Bob's labs are secure - no information leaks
- Alice and Bob control and trust devices in their labs
- Alice and Bob have free will and can **choose** their observables

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How to keep a secret

Quantum
cryptography,
randomness
and cunning
can outfox
the snoopers

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the way we were

Ancient DNA is rewriting
human prehistory
page 414

quantum physics

why it's all about me

On the physical
nature of the Now
page 421

biomedicine

make the most of mice

Better use of disease models
can save human lives
page 423

▶ nature.com/nature

27 March 2014 £10

Vol. 507, No. 7493

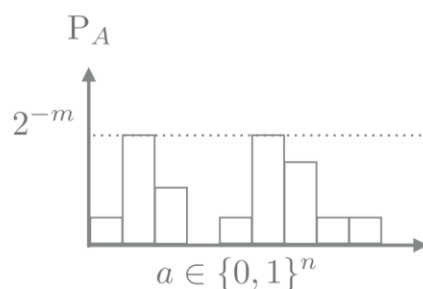


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- Can we do DIQKD with partially secret randomness – your research project 😊
- ...

How to quantify what we do not know?

Weak source of randomness



Min-entropy:

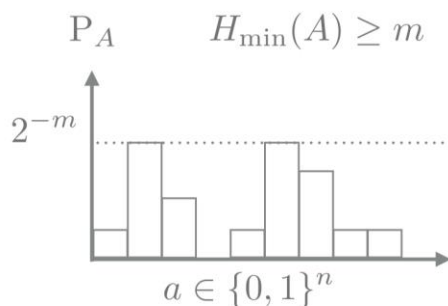
$$H_{\min}(A) = -\log \left(\max_a \Pr[a] \right)$$

$p_{\text{guess}}(A)$

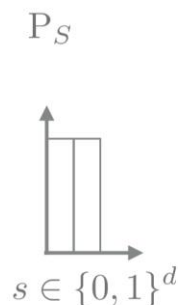
$$H_{\min}(A) \geq m :$$

$$\forall a \in \{0, 1\}^n, \quad \Pr[a] \leq 2^{-m}$$

Weak source of randomness



\times



$\xrightarrow{\text{Ext}(A, S)}$

Uniform distribution

