Independence and Conditional Aspects of Probability

Banach Center
INSTITUTE OF MATHEMATICS PAS

Faculty of Mathematics and Information Science
WARSAW UNIVERSITY OF TECHNOLOGY

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Time	Monday	Tuesday	Wednesday	Thursday	Friday
8:00–9:30	Breakfast	Breakfast	Breakfast	Breakfast	Breakfast
9:30–10:20	Adam Jakubowski	Baby Talk: Paweł Hitczenko	Małgorzata Bogdan	Rafał Latała	END
10:20–11:00	Coffee break	Coffee break	Coffee break	Coffee break	
11:00–11:30	Zbigniew S. Szewczak	Włodzimierz Bryc	Tomasz Rychlik	Adam Osękowski	
11:40–12:10	Gholamhossein Hamedani	Jacek Wesołowski	Bartosz Kołodziejek	Witold Bednorz	
12:20–12:50	Mariusz Niewęgłowski	Marek Bożejko		Rafał Łochowski	
13:00–15:00	Lunch	Lunch	Lunch	Lunch	
15:00–15:50	Katarzyna Pietruska–Pałuba	Romuald Lenczewski	Excursion	Dariusz Buraczewski	
15:50–16:30	Coffee break	Coffee break		Coffee break	
16:30–17:00	Ewa Damek	Anna Wysoczańska-Kula		Konrad Kolesko	
17:10–17:40	Agnieszka Zięba	Kamil Szpojankowski		Zahirul Hoque	
17:50–18:20	Marcin Świeca	Janusz Wysoczański			
19:00–20:00	Dinner	Dinner	Barbecue	Conference Dinner	
20:00–21:00	Baby Talk: Włodzimierz Bryc	Baby Talk: Marek Bożejko			

List of speakers

Infinitely divisible processes - how to bound them

Witold Bednorz

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There is a new result on selector type process, that solves the Talagrand conjecture. We give a slightly modified idea of the proof and discuss consequences of the result for infinitely divisible processes.

Sparse Graphical Modelling via the Sorted L^1-Norm

Małgorzata Bogdan

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Sparse graphical modelling has attained widespread attention across various academic fields. We propose two new graphical model approaches, Gslope and Tslope, which provide sparse estimates of the precision matrix by penalizing its sorted L^1-norm, and relying on Gaussian and t-student data, respectively. We provide the selections of the tuning parameters which provably control the probability of including false edges between the disjoint graph components and empirically control the False Discovery Rate for the block diagonal covariance matrices. In extensive simulation and real world analysis, the new methods are compared to other state-of-the-art sparse graphical modelling approaches. The results establish Gslope and Tslope as two new effective tools for sparse network estimation, when dealing with both Gaussian, t-student and mixture data.
This is a joint work with Riccardo Riccobello (University of Trento), Giovanni Bonaccolto (University of Enna), Philipp J. Kremer (EBS Universität für Wirtschaft und Recht), Sandra Paterlini (University of Trento) and Piotr Sobczyk (OLX group, Poznań).

Khintchine-Haagerup inequality -scalar and operator valued case.

Marek Bożejko

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In my talk I would like to present the following topics:

1. Khinchine inequality on free generators of the groups , for generators of Coxeter groups and for generalized Gaussian random variables like-q-Gaussian, boolean Gaussian,Yang-Baxter Gaussian.
2. Applications:
   a) Central Limit Theorems and relations with the best constants in the free, Fermi and q-probabilities.
   b) Von Neumann and C^*-algebras: factoriality, CBAP, non-injectivity, uniqueness of trace, and
   c) Ultracontractivity of q-Ornstein-Uhlenbeck semigroups and consequences.

Baby talk: Classical and noncommutative Khinchine inequality with optimal constants

Marek Bożejko

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The main topics of my talk are following:

1. Khintchine type inequality in $L^p, p > 2$ on Abelian and noncommutative groups: existence and the best constants.
2. Matrix-valued Khintchine inequality for Rademacher functions – F. Lust-Piquard, G.Pisier Theorem - relations with row and column operator spaces-cases of R. Schatten classes $\mathcal{S}(p)$.
3. Boolean, Fermi and Kesten Gaussian operators and optimal constants in Khintchine Type Inequalities.

Stationary measures of the Kardar-Parisi-Zhang equation and their limits

Włodzimierz Bryc

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I will overview recent results of [5] on the existence of stationary measures for the KPZ equation on an interval and [2, 4] who found two different probabilistic descriptions of the stationary measures as a Markov process and as a measure with explicit Radon-Nikodym derivative with respect to the Brownian motion.
The Markovian description leads to rigorous proofs of some of several limiting results claimed in [2]. I shall discuss how the stationary measures of the KPZ equation on $[0, L]$ behave at large scale as L goes to infinity which according to [1], depending on the normalization, should correspond to stationary measures of a hypothetical KPZ fixed point on $[0, 1]$, to stationary measure for the KPZ equation on half-line, and to stationary measures of a hypothetical KPZ fixed point on $[0, \infty)$.
The talk is based on a joint work [3] with Alexey Kuznetsov.

References

[1] Guillaume Barraquand and Ivan Corwin, *Stationary measures for the log-gamma polymer and KPZ equation in half-space*, (2022). https://arxiv.org/abs/2203.11037.

[2] Guillaume Barraquand and Pierre Le Doussal, *Steady state of the KPZ equation on an interval and Liouville quantum mechanics*. Europhysics Letters, **137**(6):61003, (2022). ArXiv preprint with Supplementary material: https://arxiv.org/abs/2105.15178.

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Random walks in a strongly sparse random environment

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The integer points of the real line are marked by the positions of a standard random walk with positive integer jumps. We assume that the set of marked sites is strongly sparse, i.e. the jumps of the random walk have infinite mean and regularly varying. We consider a nearest neighbor random walk on the set of integers having jumps +/-1 with probability 1/2 at every nonmarked site, whereas a random drift is imposed at every marked site. We will present some new limit theorems for the so defined random walk in a strongly sparse random environment. The talk will be based on a joint work with Alexander Iksanov, Piotr Dyszewski and Alexander Marynych.

Stochastic difference equation with diagonal matrices

Ewa Damek

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Let $\mathbf{A} = diag(A_1, ..., A_d)$ be a random diagonal matrix and X, B random vectors in $\mathbb{R}^d$ We assume that $\mathbf{A}$ and X are independent and

$$X \overset{in\ law}{=} \mathbf{A}X + B. \tag{1}$$

We do not assume that X and B are independent. Under appropriate (Goldie-Kesten type) conditions we prove that X is regularly varying in a nonstandard way. Namely, suppose that for every j, there is α_j such that

$$\mathbb{E}|A_j|^{\alpha_j} = 1.$$

We define

$$\delta_{t^{-1}}X = \left(t^{-1/\alpha_j}X_1, ..., t^{-1/\alpha_d}X_d\right).$$

Then the sequence of measures

$$\Lambda_t(W) = t\mathbb{P}\left(\delta_{t^{-1}}X \in W\right), \quad W \subset \mathbb{R}^d, \tag{2}$$

tends in a weak sense to a random measure Λ and $\Lambda(\mathbb{R}^d \setminus B_r(0)) < \infty$ for every ball $B_r(0)$. We also provide conditions for $\Lambda \neq 0$. For coordinates X_j, $j = 1, ..., d$ of X, (2) means

$$\lim_{t\to\infty} \mathbb{P}(\pm X_j > t)t^{\alpha_j} = c_{\pm}.$$

The most common example of (1) comes form the stochastic recurrence equation

$$\mathbf{X}_n = \mathbf{A}_n\mathbf{X}_{n-1} + \mathbf{B}_n, \quad n \in \mathbb{N}, \tag{3}$$

where $(\mathbf{A}_n, \mathbf{B}_n)$ is an i.i.d. sequence, $\mathbf{A}_n$ are $d \times d$ matrices, $\mathbf{B}_n$ are vectors and $\mathbf{X}_0$ is an initial distribution independent of the sequence $(\mathbf{A}_n, \mathbf{B}_n)$. Under mild contractivity hypotheses the sequence $\mathbf{X}_n$ converges in law to a random vector X that is the unique solution to the equation (1). In this case X is independent of $(\mathbf{A}, \mathbf{B})$.
However, there are natural examples related to random Gaussian field when X and B in (1) may be dependent. Moreover, it turns out that only independence of A and X is really essential in the proof of regular variation.

Characterizations of Probability Distributions via Concept of Sub-Independence

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Limit theorems as well as other well-known results in probability and statistics are often based on the distribution of the sums of independent (and often identically distributed) random variables rather than the joint distribution of the summands. Therefore, the full force of independence of the summands will not be required. In other words, it is the convolution of the marginal distributions which is needed, rather than the joint distribution of the summands which, in the case of independence, is the product of the marginal distributions. The concept of ub-independence, which is much weaker than that of independence, is shown to be sufficient to yield the conclusions of these theorems and results. It also provides a measure of dissociation between two random variables which is much stronger than uncorrelatedness. This is precisely the reason for the statement: "why assume independence when you can get by with sub-independence".

Randomly Growing Surfaces, Exclusion Processes and Combinatorial Tableaux

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In this talk I will give an overview of connections between models of randomly growing surfaces, simple exclusion processes, and combinatorial tableaux, currently one of the most active areas of probability theory.

Improved Estimation under Various Losses

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This study considers several alternative estimators of parameters of linear models. The estimators include unrestricted, restricted, pre-test and Stein-type estimators. The construction of the estimators and their performance are measured under the symmetric as well as asymmetric losses. Both analytical and graphical analyses of the performance of the proposed estimators are shown for superiority of one over the other(s). Finally, some future research directions are discussed for possible collaboration.

Limit theorems with conditioning: maxima and sums

Adam Jakubowski

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We begin with two recently found examples of exchangeable sequences which show that the notion of a phantom distribution function is crucial in understanding limit theorems for maxima of stationary sequences. We then examine a corresponding limit theorem for sums and recall the Principle of Conditioning - a method of derivation of limit theorems for dependent random variables. In particular we show how this method works when applied to the Lindeberg Central Theorem. We also discuss several recent applications.

Limit theorems for general branching processes

Konrad Kolesko

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For a branching random walk $\{S(u)\}_{u\in\mathbb{T}}$ with nonnegative increments and a (possibly random) function $\phi : [0,\infty) \mapsto \mathbb{R}$ we define a general branching process $\mathcal{Z}_t^\phi$ by

$$\mathcal{Z}_t^\phi := \sum_{u\in\mathbb{T}} \phi(t - S(u)).$$

Nerman, in his famous paper [1], under some mild assumptions including the existence of a Malthusian parameter $\alpha > 0$ established the strong law of large numbers:

$$e^{-\alpha t}\mathcal{Z}_t^\phi \to \int \mathbb{E}\phi(t)e^{-\alpha t}dt \cdot W \quad \text{almost surely}$$

for some non-negative random variable W that does not depend on ϕ and $\mathbb{E}W = 1$.
In the talk we present an asymptotic expansion of $\mathcal{Z}_t^\varphi$ as $t \to \infty$ up to Gaussian fluctuations. More precisely, we show that there is a constant $k \in \mathbb{N}_0$ and a function $H(t)$, a finite random linear combination of functions of the form $t^j e^{\lambda t}$ with $\alpha/2 \leq \mathrm{Re}(\lambda) < \alpha$, such that

$$\frac{\mathcal{Z}_t^\varphi - \int \mathbb{E}\phi(t)e^{-\alpha t}dt \cdot e^{\alpha t} \cdot W - H(t)}{\sqrt{t^k e^{\alpha t} W}} \xrightarrow{d} \mathcal{N}(0, \sigma^2),$$

for some $\sigma > 0$. Our result unifies and extends various earlier limit theorems for specific branching processes available in the literature.
The talk is based on a joint paper with Alexander Iksanov and Matthias Meiners.

References

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Tails of the free multiplicative convolution powers

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In the talk I will discuss the problem of the tail behavior of free multiplicative convolution powers $\mu^{\boxtimes t}$, $t \geq 1$. We consider measures μ on the positive half-line with regularly varying tail, which means that

$$\mu\big((x, +\infty)\big) \sim x^{-\alpha} L(x), \tag{4}$$

where $\alpha \in [0, \infty)$, L is a slowly varying function, and $f(x) \sim g(x)$ means that $g(x)/f(x) \to 1$ as $x \to +\infty$.
I will present a complete characterization of the behavior of the S-transform at 0^- of measures with regularly varying tails.
This result is applied to study a tail behavior of $\mu^{\boxtimes t}$ for measures as in (4). We observe an interesting phase transition in the tails between regimes $\alpha < 1$ and $\alpha > 1$.
The talk is based on a joint work with Kamil Szpojankowski (Warsaw University of Technology).

References

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Order Statistics of Log-Concave Random Vectors

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We will discuss two-sided bounds for expectations of order statistics (k-th maxima) of moduli of coordinates of centered log-concave random vectors with uncorrelated coordinates. Our bounds are exact up to multiplicative universal constants in the unconditional case for all k and in the isotropic case for $k \leq n - cn^{5/6}$. We also present two-sided estimates for expectations of sums of k largest moduli of coordinates for some classes of random vectors. Based on a joint work with Marta Strzelecka.

Motzkin cumulants

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Free cumulants are analogs of classical cumulants in the theory of free probability, in which classical independence is replaced by the concept of freeness. They are the coefficients of the R-transform of Voiculescu which plays the role of a noncommutative analog of the logarithm of the Fourier transform. From the combinatorial point of view they were defined in terms of moments by Speicher with the use of the family of lattices of noncrossing partitions. Using these lattices as well as the lattices of Motzkin paths, we introduce a hierarchy of lattices of noncrossing partitions adapted to Motzkin paths. It gives a decomposition of the combinatorics of noncrossing partitions into smaller fragments, in which depths of blocks are related to Motzkin subpaths of given Motzkin paths. Taking advantage of this hierarchy, we define path-dependent operator-valued cumulants called Motzkin cumulants and prove the corresponding Möbius inversion formula which plays the role of a decomposition of the relation between free cumulants and boolean cumulants derived by Lehner (univariate case) and Belinschi and Nica (multivariate case). More importantly, we show that it leads to a decomposition of free cumulants in terms of scalar counterparts of Motzkin cumulants.

Local times of real, self-similar processes as normalized numbers of interval crossings

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In my talk I will state a general result on a relationship between normalized numbers of interval crossings by a càdlàg path and an occupation measure, appropriately defined for this path. Using this result I will define local times of fractional Brownian motions (classically defined as densities of a relevant occupation measure) as weak limits of properly normalized numbers of interval crossings. I will also refer and discuss a similar result for càdlàg semimartingales, in particular for alpha-stable processes, and provide natural examples of deterministic paths which possess quadratic variation but no local times.

Multivariate Markovian Hawkes Processes

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A very interesting and important class of stochastic processes was introduced by Alan Hawkes in [1]. These processes, called now Hawkes processes, are meant to model self-exciting and mutually-exciting random phenomena that evolve in time. The self-exciting phenomena are modeled as univariate Hawkes processes, and the mutually-exciting phenomena are modeled as multivariate Hawkes processes. In this talk we provide some results on markovianity of the Generalized Multivariate Hawkes Processes (GMHP) introduced in our earlier papers. GMHP are multivariate marked point processes that add an important feature to the family of the (classical) multivariate Hawkes processes: they allow for explicit modelling of simultaneous occurrence of excitation events coming from different sources, i.e. caused by different coordinates of the multivariate process. It is well known that classical multivariate Hawkes processes with exponential kernels leads to some multivariate Markov processes. We provide results which goes far beyond exponential kernels and show that under some conditions on kernels the intensities of GMHP's are functions of Markov processes. Moreover we show that it is possible to compute their Laplace transform by means of system of ODE's.

References

[1] A.G. Hawkes, *Spectra of Some Self-Exciting and Mutually Exciting Point Processes*, Biometrika **58** (1971) no 1, 83–90.

[2] A. Boumezoued, *Population viewpoint on Hawkes processes*, Advances in Applied Probability, **48** (2016) no. 2, 463–480.

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Rosenthal's inequality and its extensions

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A celebrated result of Rosenthal asserts that for $p \in [2, \infty)$ there is a finite constant c_p such that the following holds. If $\xi_1, \xi_2, \ldots$ are independent, mean zero random variables in L^p, then

$$\left\| \sum_{k=1}^{n} \xi_k \right\|_p \leq c_p \left(\left\| \sum_{k=1}^{n} \xi_k \right\|_2 + \left\| \sum_{k=1}^{n} |\xi_k|^p \right\|_p^{\frac{1}{p}} \right), \quad n = 1, 2, \ldots$$

The purpose of the talk is to discuss several variants and extensions of this estimate, under various structural conditions imposed on $(\xi_k)_{k \geq 1}$.

Lifschitz tail for Anderson model driven by jump processes

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We study random Schrödinger operators based on generators of jump processes (either Lévy on $\mathbb{R}^d$, or subordinate Brownian motions on fractals). The random potential is of Anderson type: independent random variables ξ_i are placed at lattice points, then a prescribed positive profile W , with magnitude ξ_i, is attached at those points. More precisely, the operator we consider is $H^\omega = -L + V^\omega$, where L is the generator of a jump process, and V^ω is the potential given by

$$V^\omega(x) = \sum_i \xi_i(\omega)W(x,i)$$

the summation running over $i \in \mathbb{Z}^d$ in the Euclidean case, and over i from fractal lattice in the fractal case. We establish the existence of integrated density of states in these models, then we prove the Lifschitz tail behaviour at zero (exponential decay, as opposed to polynomial decay when no random potential is present). The results were obtained jointly with Kamil Kaleta (for Lévy processes on $\mathbb{R}^d$) and Hubert Balsam, Kamil Kaleta, Mariusz Olszewski (subordinate Brownian motions on nested fractals).

References

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Diagonally dependent copulae, with applications to reliability theory

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The diagonally dependent copula is one that has identical diagonal sections of all multidimensional subcopulae of the same size. The notion is a significant generalization of the exchangeable copula. We show that the diagonal dependence of the copula of the joint distribution of the component lifetimes together with the identity of the component lifetime marginal distributions are the necessary and sufficient conditions for the Samaniego representation of the distributions of the lifetime distributions of all coherent systems. The representation is a convex combination of the distributions of the consecutive failure times of system components with the combination coefficients depending merely on the system structure, and it is a crucial tool in the reliability analysis.

On relative stability for strongly mixing sequences with infinite moments

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Let $\{\xi_k\}_{k\in\mathbb{Z}}, \mathbb{Z} = \{\ldots,-1,0,1,2,\ldots,k,\ldots\}$, be a strictly stationary random sequence defined on a probability space $(\Omega,\mathcal{F},P)$ taking values on the real line $\mathbb{R}$. Set c_n to be a sequence of real numbers and $S_n = \sum_{k=1}^{n} \xi_k, n \in \mathbb{N}, \mathbb{N} = \{1,2,\ldots,n,\ldots\}$. We call $\{\xi_k\}$ relatively stable if $c_n^{-1}S_n \to 1$, in probability. Relative stability plays important role in the study of CLT for martingales or GARCH processes (see e.g. [4, 5, 7]).
Let $\varphi(n) = \sup\{|P(B|A) - P(B)|; A \in \mathcal{F}_{-\infty}^{0}, B \in \mathcal{F}_n^{\infty}\}$, where $\mathcal{F}_k^m = \sigma(\{\xi_i; k \le i \le m\})$. We say that $\{\xi_k\}$ is uniformly strong mixing if $\varphi_n \to_n 0$. It is well-known that uniformly strong mixing sequence $\{\xi_n^2\}$ is relatively stable iff $E[\xi_0^2 I_{[\xi_0 \le x]}]$ varies slowly. Here $c_n = b_n^2$, where b_n satisfies the asymptotic relation $b_n^2 \sim nE[\xi_0^2 I_{[|\xi_0|\le b_n]}]$ (cf. [2, 3, 6]).
Let $\alpha(n) = \sup\{|P(B\cap A) - P(A)P(B)|; A \in \mathcal{F}_{-\infty}^{0}, B \in \mathcal{F}_n^{\infty}\}$. We say that $\{\xi_k\}$ is strongly mixing if $\alpha_n \to_n 0$.

Theorem. *There exists strongly mixing sequence $\{\xi_k^2\}$ with $E[\xi_0^2 I_{[|\xi_0|\le x]}]$ slowly varying, $E[\xi_0^2] = \infty$, which is not relatively stable under normalizing $c_n = b_n^2$*

In view of Theorem 2 in [5] we have the following (see also [1, 6]).

Corollary. *There exists strongly mixing (martingale differences) sequence $\{X_k\}_{k\in\mathbb{Z}}$ with $E[X_0^2 I_{[|X_0|\le x]}]$ slowly varying, $E[X_0^2] = \infty$, for which CLT fails to hold under normalizing b_n.*

This is the joint research with A. Jakubowski.

References

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Conditional expectations and subordination of (some) polynomials in free random variables via Boolean cumulants

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In this joint talk the goal is to establish Boolean cumulants as an effective tool for the calculation conditional expectations and distributions of polynomials in free random variables. In particular using Boolean cumulants one can obtain "subordination" equations for the resolvents of arbitrary polynomials in free random variables, generalizing the case of free additive and multiplicative free convolution previously established by Biane and Voiculescu.
The talk is divided into two parts:

1. In the first part we present a new (more algebraic) approach to our previous results from [1], which relate conditional expectations and Boolean cumulants.
2. In the second part we will explain how to compute the distribution of any polynomial in free random variables using the algebraic relations obtained in first part.

Talk is based on a joint, on going work with F. Lehner (TU Graz).

References

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The Matsumoto-Yor property in free probability

Marcin Świeca

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Let X and Y be independent, positive and nondegenerate random variables and let $U = \frac{1}{X+Y}$ and $V = \frac{1}{X} - \frac{1}{X+Y}$. Then U and V are independent if and only if X has the Generalized Inverse Gaussian distribution $GIG(-p, a, b)$ and Y has the Gamma distribution $G(p, a)$ where $a, b, p > 0$. This property has been generalized to the framework of free probability and characterizes Marchenko-Pastur and free Generalized Inverse Gaussian laws. In my talk I will discuss regression versions of this characterization. The talk is based on my preprint arXiv:2109.12545

Approaching the triple point in ASEPs with open boundaries

Jacek Wesołowski

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ASEP is a model of evolution of particles on sites $1, \ldots, n$. The particles can: (1) jump to the neighbor site (if empty) with rate 1 to the right and rate $q(< 1)$ to the left; (2) enter the system at site 1 or n (if empty) at rate α or δ; (3) exit the system at site 1 or n (if non-empty) at rate γ or β. The triple point in the parameter space of ASEP is the point, where the three regions: "maximal current", "low density" and "high density", of the ASEP parameter space meet. We are interested in so-called height function under the stationary measure of ASEP. We derive its in-distribution-limit when parameters approach the triple point as the number of sites n tends to infinity, while q is kept fixed. This is a joint work with Wlodek Bryc and Yizao Wang from Univ. of Cincinnati.

Hunt formula for $SU_q(n)$

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The well-known Lévy-Khintchine formula states that generators of Lévy processes on $\mathbb{R}^n$ are combinations of continuous (or Gaussian) parts and jump parts. Its generalization, the Hunt formula, gives a similar description for Lévy procesess on Lie groups: theirs generators are sums of at most second-order terms (Gaussian part) and limits of Poisson type generators. Going further with a generalization, to noncommutative Lévy processes on $*$-bialgebras, we still can characterize Lévy processes by theirs generators, but an analoguous decomposition might not exist. In my talk, I will sketch different situations that can happen and, in particular, will discuss the case of $SU_q(N)$.
This is the joint work with U. Franz, M. Lindsay and M. Skiede.

Law of Small Numbers for Random Variables indexed by Positive Symmetric Cones

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We present an analogue of the classical Law of Small Numbers, formulated for the notion of bm-independence, where the random variables are indexed by elements of positive symmetric cones in Euclidean spaces, including R^d_+, the Lorentz cones in Minkowski spacetime and positive definite real symmetric matrices. The geometry of the cones plays a significant role in the study as well as the combinatorics of bm-ordered partitions. The talk is based on the joint paper with Lahcen Oussi (University of Wrocław).

Infinitesimal generators of quadratic harnesses

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Quadratic harnesses are Markov polynomial processes with linear conditional expectations and quadratic conditional variances with respect to the past-future filtrations, which are typically determined by five numerical constants which appear in the expression for the conditional variance. It turns out that infinitesimal generators of such processes are identified through a solution of a q-commutation equation in the algebra of infinite sequences of polynomials in one variable.
In my presentation, I will show that this solution is a special element whose coordinates satisfy a three terms recurrence and define a system of orthogonal polynomials. The respective orthogonality measure uniquely determines the infinitesimal generator (acting on polynomials or bounded functions with bounded second derivative) as an integro-differential operator, where the integration is with respect to this measure.