On geometric complexity of Julia sets - IV

Będlewo & online (hybrid), 14-19.08.2022

Abstracts of talks

Slowly recurrent Collet-Eckmann maps on the Riemann sphere
Magnus Aspenberg

Conjecturally, almost every rational map is either hyperbolic or non-uniformly expanding, i.e. satisfies the Collet-Eckmann condition (CE). In this talk I will present two results on perturbations of CE-maps, where the main novelty is to allow the critical set to be recurrent at a slow rate (slowly recurrent maps). Suppose f is such a slowly recurrent CE-map. If the Julia set is the whole sphere, then f is a Lebesgue density point of CE-maps, and if the Julia set is not the whole sphere, then f is a Lebesgue density point of hyperbolic maps. The last part is a joint work with M. Bylund and W. Cui.

Typical absolute continuity for one-parameter families of dynamically defined measures
Balázs Bárány

Consider a one-parameter family of iterated function systems on the interval and a family of measures on the corresponding symbolic space, depending on the same parameter. There are several naturally occurring measures in this class, like invariant measures for systems with place-dependent probabilities and equilibrium measures for hyperbolic iterated function systems. Assuming that the IFS satisfies the transversality condition and the measures depend on the parameter regularly enough, we obtain results on the dimension and the absolute continuity of the projected measure for almost every parameter. The talk is based on the joint work with Károly Simon, Boris Solomyak and Adam Śpiewak.

Connected McMullen-like Julia sets via singular perturbations
Jordi Canela

Singular perturbations of rational maps were introduced by McMullen. He studied the rational family $M_{\lambda,m,n}(z) = z^m + \lambda/z^n$ in order to provide an example of a rational map with a buried Julia component. The corresponding Julia set is a Cantor set of quasicircles.

In some cases, holomorphic families of rational maps R_a may have degeneracy parameters, i.e. parameters for which the degree of the map R_a decreases. For instance, $\lambda = 0$ is a degeneracy parameter of $M_{\lambda,m,n}$. In this talk we will explain how singular perturbations can be used to understand the dynamics of families with degeneracy parameters. More specifically, we will show how to relate the dynamics of a family of rational maps R_a obtained from Chebyshev-Halley root finding algorithms with the dynamics of the family $M_\lambda(z) = z^4 + \lambda/z^2$. In particular, we will prove that if $|a|$ is small enough, $a \neq 0$, then the Julia set of R_a is connected and contains an invariant Cantor set of quasicircles.

This is a joint work with Antonio Garijo and Pascale Roesch.

Upper bounds for Hausdorff dimension of Julia sets (revisited)
Neil Dobbs

In joint work with Graczyk and Mihalache, we showed upper and lower bounds on the dimension of quadratic Julia sets for real parameters $-2+\epsilon$ near -2. I will recall these results and indicate how the upper bound (for a positive measure set of parameters) can be improved to $1 + C(\log \epsilon^{-1})\sqrt{\epsilon}$, approaching the general lower bound $1 + C\sqrt{\epsilon}$. The proofs use a mixture of techniques: parameter exclusion, orbital estimates, conformal measures etc.

'Back-box-mappings' and rigidity of rational maps
Kostiantyn Drach

We will discuss a generalized renormalization concept called 'complex box mapping' and its application to rigidity of rational maps. Complex box mappings arise naturally as first return maps to well-chosen domains and generalize the classical notion of polynomial-like mapping. In the first half of the talk, we will present results about - rigidity (qc rigidity, absence of invariant line fields), - ergodicity (count of the ergodic components), - expansivity (a Mañé-type result), for complex box mappings. In the second half of the talk, we will illustrate when and how the machinery of box mappings can be applied (almost as a 'black box') to conclude analogues results for certain multidimensional families of rational maps.

Thurston-type realization theorem for the boundaries of disjoint type hyperbolic components
Dzmitry Dudko

We consider hyperbolic components of disjoint type: every grand orbit of Fatou components contains a unique simple critical point. Such components are parametrized by the multipliers of attracting periodic cycles. At the boundary, some of the cycles become neutral. We will describe the boundaries of disjoint type hyperbolic components by characterizing the realized sets of non-repelling multipliers. In the Sierpinski case, all boundary parameters are realized; i.e., Sierpinski hyperbolic components of disjoint type are bounded. Joint work with Yusheng Luo.

Combinatorial structure of the renormalization attractor for multicritical circle maps
Igors Gorbovickis

We will describe the combinatorial structure of the attractor of renormalization for analytic multicritical circle maps whose rotation numbers are of bounded type. In particular, in the special case of unicritical maps it is known that the renormalization operator $\mathcal{R}$ is bijective on the attractor, and for any two distinct maps f and g in the attractor, there exists an integer $n \geq 0$, such that the maps $\mathcal{R}^{-n}f$ and $\mathcal{R}^{-n}g$ are combinatorially different. We prove that this does not hold when the number of critical points is greater than two. More specifically, we show that in this case renormalization is still bijective on the attractor, but there exist two distinct maps f and g in the attractor, such that for any integer n, the maps $\mathcal{R}^{-n}f$ and $\mathcal{R}^{-n}g$ have the same combinatorics. This is joint work with Michael Yampolsky.

Dynamics on the boundary of Fatou components
Anna Jové-Campabadal

We study boundary orbits of unbounded invariant Fatou components of transcendental entire functions. The fact that infinity is an essential singularity of the function gives interesting topological and dynamical properties. In particular, we will present new conditions which implies that periodic boundary points and escaping boundary points are dense in the boundary. This is based on joint work in progress with N. Fagella.

Around a formula for the speed of multipliers
Genadi Levin

Given a rational map f and its periodic orbit O with a multiplier $\rho \neq 0, 1$, the formula (which was proven earlier) relates, on the one hand, the action of $I - T$ on a certain explicit rational function (quadratic differential) with double zeros at O and, on the other, the gradient of ρ as a function of critical values of f. Here T is the Ruelle-Thurston pushforward operator associated to f. In the talk, we discuss some applications of the formula. Work in progress.

A priori bounds and degeneration of Herman rings
Willie Rush Lim

Given a rational map, a Herman curve is an invariant Jordan curve that is not contained in the closure of a rotation domain and on which the map is conjugate to a rigid rotation. I would like to give a strong affirmative answer to a question by A. Eremenko at 'On Geometric Complexity of Julia Sets II' on the existence of Herman curves that are non-trivial, i.e. do not come from a Blaschke product. The strategy is as follows. By adapting the near-degenerate regime, we show that the boundaries of Herman rings of bounded type rotation number and of the simplest configuration are quasicircles with dilatation depending only on the degree and the rotation number. Then, we study the limits of degenerating Herman rings and construct general examples of rational maps with a non-trivial Herman curve of arbitrary degree and combinatorics.

Conformal structures on stochastic subdivision rules
Peter Lin

We consider 'conformal' parameterizations of random fractal spaces X arising as limits of certain stochastic subdivision rules. One motivation comes from the field of random geometry, where it is an important and difficult problem to understand this parameterization when X arises from limits of random planar maps. Deterministic versions of our model, and the analogous questions relating to conformal parameterizations, are closely related to Thurston's topological characterization of rational maps. In all the settings mentioned above, a key difficulty is in proving that the fractal approximations do not degenerate in the complex analytic sense. We overcome this difficulty in our setting by proving a contraction inequality for probabilistic iteration on a variant of the universal Teichmuller space. This inequality also provides a different perspective on random quasiconformal map models considered by Astala-Rohde-Saksman-Tao and Ivrii-Markovic.

Generating holomorphic functions with critical orbit relation
Khudoyor Mamayusupov

We proposed an algorithmic method for generating holomorphic functions with critical orbit relations. The method is based on the observation that the critical points solve a quadratic equation and thus the powers of critical points can be reduced to an affine function. We demonstrate our method for a case of cubic polynomials and some families of quadratic and cubic rational functions. Irreducibility of obtained curves is left as an open problem. Our approach also works for generating critical orbit relations in any families of rational functions with active critical points.

Wandering domains in transcendental dynamics: topology and dynamics
David Martí-Pete

For a transcendental entire or meromorphic function, the Fatou set is the largest open set on which its iterates are defined and form a normal family. A wandering domain is a connected component of the Fatou set which is not eventually periodic. Wandering domains, which do not exist for rational maps, play an important role in transcendental dynamics and in the last decade there has been a resurgence in their interest.

Wandering domains are very diverse in terms of both their topology and their dynamics. Recently, Boc Thaler proved the surprising result that every bounded regular domain such that its closure has a connected complement is the wandering domain of some transcendental entire function. Inspired by this result, together with Rempe and Waterman, we were able to obtain wandering domains that form Lakes of Wada.

In this talk, I will describe the main topological and dynamical properties of wandering domains (and their boundaries) and give an overview of the current open questions.

Symmetries of Julia Sets
Sergiy Merenkov

I plan to discuss groups acting on Julia sets of various topological types, such as Sierpiński carpets, gaskets, and tree-like sets. The actions are quasisymmetric and exhibit a range of rigidity/flexibility phenomena, from topological rigidity in the case of gaskets to quasisymmetric rigidity of carpets, and to flexibility of tree-like sets. The talk is based on several joint projects with Mario Bonk, Russell Lodge, Misha Lyubich, Sabya Mukherjee, and Dimitrios Ntalampekos.

David homeomorphisms in analysis and dynamics
Sabyasachi Mukherjee

We will discuss a David extension theorem for circle homeomorphisms conjugating hyperbolic maps to parabolic ones. Various applications of this extension result to problems in conformal welding, removability and combinations of conformal dynamical systems (as maps as well as correspondences) will be sketched.

Based on joint works with Mikhail Lyubich, Jacob Mazor, Sergiy Merenkov, and Dimitrios Ntalampekos.

Definitions of quasiconformality and exceptional sets
Dimitrios Ntalampekos

In this talk I will present a new generalized metric definition of quasiconformality for Euclidean space, requiring that at each point there exists a sequence of uncentered open sets with bounded eccentricity shrinking to that point such that the images also have bounded eccentricity. The classical metric definition requires instead that infinitesimal balls are mapped to sets of bounded eccentricity. Then I will discuss exceptional or else removable sets for that definition, introducing the new class of CNED sets (countably negligible for extremal distance). These sets are removable and reconcile several of the previously known classes of removable sets.

The maximum modulus set of an entire function
Leticia Pardo-Simón

The set of points where an entire function achieves its maximum modulus is known as the maximum modulus set, and usually consists of a collection of disjoint analytic curves. In this talk, we discuss recent progress on the description of the features that this set might exhibit. Namely, on the existence of discontinuities, singleton components, and on its structure near the origin and near infinity. This is based on joint work with D. Sixsmith, V. Evdoridou and A. Glücksam.

A counterexample to Eremenko's Conjecture
Lasse Rempe

I shall speak within my lecture
about an interesting conjecture
of Eremenko from a fine
paper of 1989.

He asked if each escaping point
can to infinity be joined
using a connected shape
all points of which themselves escape.

Although quite simple it appears,
this question has for many years
caused me and others some despair,
sleepless nights and greying hair.

Through our intense investigation
of transcendental iteration,
much progress was indeed obtained,
but the conjecture, it remained -

till now! By work with Waterman
and Martí-Pete, now I can
describe to you, within my lecture,
a counterexample to Eremenko's Conjecture.

The negative Multibrot sets
Pascale Roesch

This is a joint work with Bastien Rossetti. In this talk we give an overview of properties of the bifurcation locus and hyperbolic components of the one parameter family $f_c(z) = z^{-d} + c$. We will concentrate en de degree 2 case for this talk.

Quantization dimension for fractals of overlapping construction
Károly Simon

Joint with Mrinal Kanti Roychowdhury. The goal of quantization is to approximate a continuous Borel probability measure by measures whose support consists of finitely many points. It has broad application in signal processing, telecommunications, data compression, image processing and cluster analysis.

For a given continuous Borel probability measure μ the quantization error $e_{n,r}(\mu)$ (where $r > 0$ is a parameter) is the distance (in Wasserstein-Kantorovitc L_r metric) between μ the and its best approximation by those discrete measures which are supported by at most n points. The quantization dimension $D_r(\mu)$ measures the asymptotic rate at which $e_{n,r}(\mu)$ goes to zero.

Namely,

$$\log e_{n,r}(\mu) \sim \log \left(\frac{1}{n}\right)^{1/D_r(\mu)}.$$

Twenty years ago, Graf and Luschgy computed the quantization dimension for self-similar measures in the cases when the underlying self-similar Iterated Function Systems satisfy the Open Set Condition (that is the cylinder sets are well separated). The problem of computing the quantization dimension for self-similar measures with essential overlaps is widely open.

In this talk we consider a self-similar IFS with heavy overlaps between the cylinder sets and we compute the quantization dimension for the self-similar measures.

Core entropy along the Mandelbrot set and Thurston's "Master teapot"
Giulio Tiozzo

The notion of core entropy was introduced by W. Thurston by taking the entropy of the restriction of a complex quadratic polynomial to its Hubbard tree. This function varies wildly as the parameter varies, reflecting the topological complexity of the Mandelbrot set.

Moreover, Thurston also defined the "master teapot", a fractal set obtained by considering for each postcritically finite real quadratic polynomial the Galois conjugates of the entropy.

In the talk, we will discuss generalizations of this fractal from real to complex polynomials. In particular, we will define a "master teapot" for each vein the Mandelbrot set, discuss continuity properties of the core entropy, and use it to prove geometric properties of the master teapot.

Joint with Kathryn Lindsey and Chenxi Wu.

A new approach to computing the Hausdorff dimension
Polina Vytnova

During the UK lockdowns Mark Pollicott and I spent the time computing Hausdorff dimension of certain fractal subsets of the interval. We developed a new and remarkably powerful method to handle sets of very complex structure, that appear in problems from the number theory. I will discuss how this method can be adapted to estimate the dimension of some Julia sets. Based on a joint work with Mark Pollicott, Julia Slipantschuk, and Caroline Wormell.

Rational maps with smooth degenerate Herman rings
Fei Yang

We prove the existence of rational maps having smooth degenerate Herman rings. This answers a question of Eremenko affirmatively. The proof is based on the construction of smooth Siegel disks by Avila, Buff and Chéritat as well as the classical Siegel-to-Herman quasiconformal surgery. A crucial ingredient in the proof is the surgery's continuity, which relies on the control of the loss of the area of quadratic filled-in Julia sets by Buff and Chéritat. As a by-product, we prove the existence of rational maps having a nowhere dense Julia set of positive area for which these maps have no irrationally indifferent periodic points, no Herman rings, and are not renormalizable.

Infinitely Renormalizable Critical Points in Dimension Two
Jonguk Yang

Hénon maps provide the simplest non-trivial two-dimensional analogs of quadratic polynomials. However, despite being one of the best studied class of examples in the field, the dynamics of these maps still remain a wide open area of research. A particularly prominent difficulty is to identify what, if anything, plays the role of the critical point.

In this talk, we consider infinitely renormalizable real Hénon maps whose renormalizations converge to one-dimensional limits. Then using a natural generalization of critical points to higher dimensions, we show that these maps are "uni-critical," and that the existence of the critical orbits completely describe the non-uniformity of partial hyperbolicity for these systems.

This is based on joint work with S. Crovisier, M. Lyubich and E. Pujals.

Hausdorff measure of limit sets of conformal iterated function systems

Anna Zdunik

Let $G = (g_i)_{i \in I}$ be a (finite or infinite) conformal iterated function system. If G is regular enough then Bowen's formula determines the Hausdorff dimension of its limit set, and (again under some regularity condition) it is known that the Hausdorff measure of the limit set evaluated at its Hausdorff dimension is positive and finite. It is also known that in analytically parametrized (sufficiently regular) systems $\lambda \mapsto G_\lambda$ the Hausdorff dimension depends analytically on the parameter λ. I will review and discuss some analogous questions about the numerical value of the Hausdorff measure of the limit set. The talk is based on joint works with Mariusz Urbański and Rafał Tryniecki.