Book of Abstracts

NUMERICAL ANALYSIS AND APPLICATIONS OF SDEs

September 25 - October 1, 2022

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Plenary Talks

Andrea Barth

University of Stuttgart

On stochastic transport with Levy noise

Semilinear hyperbolic stochastic partial differential equations have various applications in the natural and engineering sciences. From a modeling point of view the Gaussian setting may be too restrictive, since applications in mathematical finance and phenomena such as porous media or pollution models indicate an influence of noise of a different nature. In order to capture temporal discontinuities and allow for heavy-tailed distributions, Hilbert space-valued Lévy processes (or Lévy fields) as driving noise terms are considered. The numerical discretization of the corresponding SPDE involves several difficulties: Low spatial and temporal regularity of the solution to the problem entails slow convergence rates and instabilities for space/time-discretization schemes. Furthermore, the Lévy process admits values in a possibly infinite-dimensional Hilbert space, hence projections onto a finite-dimensional subspace for each discrete point in time are necessary. Finally, unbiased sampling from the resulting Lévy field may not be possible. We introduce a novel fully discrete approximation scheme that addresses all of these aspects.

Annika Lang

Chalmers University of Gothenburg

Stochastic partial differential equations and random fields on Riemannian manifolds

Random fields on manifolds can be used as building blocks for solutions to stochastic partial differential equations or they can be described by stochastic partial differential equations. In this talk I present recent developments in numerical approximations of random fields and solutions to stochastic partial differential equations on manifolds and connect the two.

Thomas Müller-Gronbach

University of Passau

On the complexity of strong approximation of SDEs with a Non-Lipschitz drift coefficient

We study pathwise approximation of an SDE at a single time or globally on a finite time interval based on finitely many (sequential) evaluations of the driving Brownian motion. We focus on the case when the drift coefficient is not Lipschitz continuous and discuss recent results on corresponding upper and lower error bounds.

Sotiros Sabanis

University of Edinburgh, National Technical University of Athens, Alan Turing Institute

Euler-Krylov polygonal schemes and new stochastic optimizers in deep learning

A new form of Euler's polygonal approximations, one which allows coefficients to depend directly on the step size, proved extremely useful in dealing with the case of superlinear coefficients. As this form was first highlighted in [1] and [2], we call the resulting approximations as Euler-Krylov polygonal schemes. They were used initially to simplify proofs and extend results on existence of SDEs with monotone coefficients. Since then, they have been applied in several other directions, including new explicit numerical schemes for SDEs with superlinear coefficients, see e.g. [3], [4] and references therein, even in the presence of discontinuities, see [5], as well as in the construction of MCMC algorithms, see e.g. [6] and [7]. More recently, this new form of Euler's polygonal approximations yielded new, stochastic (adaptive) optimization algorithms with superior performance, in many cases of training artificial neural networks, than other leading optimization algorithms such Adam, see [8] and [9]. We will review key developments of this new methodology, in particular with regards to its application in deep learning and compare empirically with the performance of adam and adam-like stochastic optimizers.

References:

[1] N. V. Krylov. Extremal properties of the solutions of stochastic equations. Theory of Probability and its Applications. 29(2):205–217, 1985.

[2] N. V. Krylov. A simple proof of the existence of a solution to the It^o's equation with monotone coefficients. Theory of Probability and its Applications. 35(3):583–587, 1990.

[3] S. Sabanis. Euler approximations with varying coefficients: the case of superlinearly growing diffusion coefficients. Annals of Applied Probability. 26(4), 2083–2105, 2016.

[4] S. Sabanis and Y. Zhang. On explicit order 1.5 approximations with varying coefficients: the case of super-linear diffusion coefficients. Journal of Complexity. 50, 84–115, 2019.

[5] T. M.-Gronbach, S. Sabanis and L. Yaroslavtseva. Existence, uniqueness and approximation of solutions of SDEs with superlinear coefficients in the presence of discontinuities of the drift coefficient. arXiv preprint arXiv:2204.02343, 2022

[6] N.Brosse, A. Durmus, 'E. Moulines, and S.Sabanis. The tamed unadjusted Langevin algorithm. Stochastic Processes and their Applications. 129(10):3638-3663, 2019.

[7] S. Sabanis and Y. Zhang. Higher order Langevin Monte Carlo algorithm. Electronic Journal of Statistics. 13(2):3805–3850, 2019.

[8] A. Lovas, I. Lytas, M. Rasonyi, and S. Sabanis. Taming neural networks with tusla: Non-convex learning via adaptive stochastic gradient langevin algorithms. arXiv preprint arXiv:2006.14514, 2020.

[9] D.-Y. Lim and S. Sabanis Polygonal Unadjusted Langevin Algorithms: Creating stable and efficient adaptive algorithms for neural networks. arXiv preprint arXiv: arXiv:2105.13937, 2021.

Contributed Talks

Evelyn Buckwar

Johannes Kepler University Linz

A couple of ideas on splitting methods for SDEs

We discuss developing splitting methods for stochastic differential equations. Splitting methods are a well-known type of numerical methods in the context of Geometric Numerical Integration of ordinary differential equations, in particular they are known to be structure preserving schemes in various situations. Extensions of these methods to the case of stochastic differential equations exist for considerable time already and they currently appear to become quite popular. In this talk I will present examples illustrating some benefits of splitting methods for SDEs. Illustrative examples include SDEs employed in neuroscience and chemical kinetics.

Oleg Butkovsky

Weierstrass Institute Berlin

Strong rate of convergence of the Euler scheme for SDEs with irregular drift driven by Levy noise

We study the strong rate of convergence of the Euler–Maruyama scheme for a multidimensional stochastic differential equation (SDE)

$$dX_t = b(X_t)dt + dL_t,$$

with irregular β-Hölder drift, $\beta > 0$, driven by a Lévy process with exponent $\alpha \in (0, 2]$. For $\alpha \in [2/3, 2]$ we obtain strong L_p and almost sure convergence rates in the whole range $\beta > 1 - \alpha/2$, where the SDE is known to be strongly well–posed. This significantly improves the current state of the art both in terms of convergence rate and the range of α. In particular, the obtained convergence rate does not deteriorate for large p and is always at least $n^{-1/2}$; this allowed us to show for the first time that the the Euler–Maruyama scheme for such SDEs converges almost surely and obtain explicit convergence rate. Furthermore, our results are new even in the case of smooth drifts. Our technique is based on a new extension of the stochastic sewing arguments.

References:

[1] A. Bcdef: An interesting paper on Operator Theory, Integral Equations and Operator Theory, 1 (1927),372–391.

[2] X. Yzw: The standard book on Linear Algebra. Springer Verlag, 1999.

David Cohen

Chalmers University of Technology, University of Gothenburg

Splitting integrators for stochastic Lie–Poisson systems

We study stochastic Poisson integrators for a class of stochastic Poisson systems driven by Stratonovich noise. For this purpose, we propose explicit stochastic Poisson integrators based on a splitting strategy, and analyse their qualitative and quantitative properties: preservation of Casimir functions and strong and weak rates of convergence. Illustrations of these properties for stochastically perturbed Maxwell–Bloch, rigid body and sine–Euler equations will be provided.

Sonja Cox

University of Amsterdam

ℓ^∞ and Hölder bounds for vector-valued stochastic integrals

We have obtained upper bounds for the ℓ^∞-norm of vector-valued stochastic integrals; more specifically, we have shown that there exists a constant C such that for every $p \in [1, \infty)$, every H-cylindrical Brownian motion W, and every sequence $(L^{(k)})_{k\in\mathbb{N}}$ of H-valued adapted stochastic processes we have

$$\left\| \sup_{k\in\mathbb{N}, T\geq 0} \frac{\left| \int_0^T \langle L_t^{(k)}, dW_t \rangle_H \right|}{\sqrt{1+\log(k)}} \right\|_{L^p(\Omega)} \leq C\sqrt{p} \left\| \sup_{k\in\mathbb{N}} \left(\int_0^\infty \|L_t^{(k)}\|_H^2 dt \right)^{\frac{1}{2}} \right\|_{L^p(\Omega)}. \tag{1}$$

Using a variation on Ciesielski's isomorphism, we use this to obtain bounds on Hölder norms for function-valued stochastic integrals. I will briefly discuss how we obtain these results, in what sense they are sharp, and discuss possible applications.

Konstantinos Dareiotis

University of Leeds

Regularisation of differential equations by multiplicative fractional noises

Differential equations (DEs) perturbed by multiplicative fractional Brownian motions are considered. Depending on the value of the Hurst parameter H, the resulting equation is pathwise viewed as an ordinary DE ($H > 1$), Young DE ($H \in (1/2, 1)$) or rough DE ($H \in (1/3, 1/2)$). In all three regimes we show regularisation by noise phenomena by proving the strongest kind of well-posedness with irregular drift: strong existence and path-by-path uniqueness. In the Young and smooth regime $H > 1/2$ the condition on the drift coefficient is optimal in the sense that it agrees with the one known for the additive case. In the rough regime $H \in (1/3, 1/2)$ we assume positive but arbitrarily small drift regularity for strong well-posedness, while for distributional drift we obtain weak existence.

Noufel Frikha

Paris Citè University

Well-posedness of McKean-Vlasov SDEs, related PDE on the Wasserstein space and some new quantitative estimates for propagation of chaos

During this talk, I will present some recent results on the well-posedness in the weak and strong sense for some non-linear diffusion processes in the sense of McKean-Vlasov which go beyond those derived from the standard Cauchy-Lipschitz theory. Under the uniform ellipticity assumption, I will show how the underlying noise regularizes the equation and allows to prove that the transition density exists and is smooth, especially in the measure direction, even if the coefficients of the dynamics are irregular. Such smoothing properties then in turn allow to establish the existence and uniqueness for the Cauchy problem associated to a backward Kolmogorov PDE stated on the Wasserstein space with irregular terminal condition and source term. This PDE on an infinite dimensional space plays a key role in order to derive new quantitative estimates of propagation of chaos for the mean-field approximation by the associated system of interacting particles. This presentation is based on the two recent works [1] and [2] in collaboration with P.-E. Chaudru de Raynal (Université Nantes).

References:

[1] P.E. Chaudru de Raynal and N. Frikha: Well-posedness for some non-linear SDEs and related PDE on the Wasserstein space, *Journal de Mathématiques Pures et Appliquées*, 159 (2022), 1–167.

[2] P.E. Chaudru de Raynal and N. Frikha:From the backward Kolmogorov PDE on the Wasserstein space to propagation of chaos for McKean-Vlasov SDEs, *Journal de Mathématiques Pures et Appliquées*, 156 (2022), 1–124.

Mate Gerencser

Vienna University of Technology

Full discretisation of reaction-diffusion equations driven by space-time white noise

We consider the space-time explicit finite difference approximation of $1 + 1$-dimensional semilinear SPDEs. Classical results of Gy¨ongy and Davie-Gaines established, in the case of Lipschitz semilinearity, both upper and lower bounds of order $1/2$ on the strong rate of convergence (w.r.t. the parabolic scaling). We show that the optimal rate $1/2$ holds in fact without any regularity assumption on the nonlinearity, crucially leveraging the regularising effects of the noise. This is joint work with Oleg Butkovsky and Konstantinos Dareiotis. Time allowing, we then discuss the case of Allen-Cahn equation. Here strong rate $1/2$ has been previously obtained, so it would appear that there is nothing to prove. However, we show how one can employ ideas from singular SPDEs to obtain some new results. This is joint work in progress with Harprit Singh.

Stefan Heinrich

University of Kaiserslautern

On the complexity of stochastic integration

We continue the study of stochastic integration in the framework of information-based complexity theory (IBC). After an introduction into basic notions of IBC for stochastic problems we first analyze stochastic integration in a general Banach space context. Then we specify the considerations to stochastic functions with values in various function spaces. We provide and analyze deterministic and randomized algorithms and prove matching lower bounds in both IBC settings, thus determining the complexities. The study includes function spaces of low smoothness, which exhibit some pecularities. Our results extend those of [1] and [2].

References:

[1] Monika Eisenmann, Raphael Kruse: Two quadrature rules for stochastic It^o-integrals with fractional Sobolev regularity, Commun. Math. Sci. 16, No. 8 (2018), 2125–2140, see also arXiv:1712.08152

[2] S. Heinrich: Complexity of stochastic integration in Sobolev classes, J. Math. Anal. Appl. 476 (2019),177–195.

Martin Hutzenthaler

University of Duisburg-Essen

Strong convergence rate for stochastic Burgers equations

Subject of this talk are strong convergence rates on the whole probability space for explicit full-discrete approximations of stochastic Burgers equations with multiplicative trace-class noise. The key step in our proof is to establish uniform exponential moment estimates for the numerical approximations.

References:

[1] A. Bcdef: An interesting paper on Operator Theory, Integral Equations and Operator Theory, 1 (1927), 372–391.

[2] X. Yzw: The standard book on Linear Algebra. Springer Verlag, 1999.

Andrzej Kałuza

AGH University of Science and Technology

Optimal global approximation of systems of jump-diffusion SDEs on equidistant mesh

In presentation, we focused on the results for a strong global approximation of systems of stochastic differential equations (SDE) of the following form

$$\begin{cases} dX(t) = & a(t, X(t))dt + b(t, X(t))dW(t) + c(t, X(t-))dN(t), t \in [0, T] \\ X(0) = & x_0, \end{cases} \tag{2}$$

where $x_0 \in \mathbb{R}^d$, functions $a, c : [0, T] \times \mathbb{R}^d \to \mathbb{R}^d$, $b : [0, T] \times \mathbb{R}^d \to \mathbb{R}^{d \times m_W}$ satisfy some regularity conditions, $W = W(t)_{\in[0,T]}$ is m_W dimensional Wiener process and $N = N(t)_{t\in[0,T]}$ is non-homogeneous Poisson process with the intensity function $\lambda = \lambda(t) > 0$. We also impose suitable commutativity conditions on diffusion and jump coefficients. We provide a construction of two algorithms for approximation of the system of the jump-diffusion SDEs. Both methods are based on the classical Milstein steps and use equidistant discretization points. We construct implementable algorithm and their derivative-free version. We show that constructed methods are asymptotically optimal in the class of methods that use equidistant discretization of the interval $[0, T]$. Moreover, we show numerical experiments performed for some test equations, which confirm theoretical properties.

References:

[1] Kałuza, A., ˙ Optimal global approximation of systems of jump-diffusion SDEs on equidistant mesh, Applied Numerical Mathematics, 179 (2022), 1-26.

[2] Kałuza, A., ˙ Optimal algorithms for solving stochastic initial-value problems with jumps, doctoral thesis.

Mihály Kovács

Chalmers University, Pázmány Péter Catholic University, Budapest University of Technology and Economics

Some approximation results for mild solutions of stochastic fractional order evolution equations driven by Gaussian noise

We investigate the quality of space approximations of a class of stochastic integral equations of convolution type with Gaussian noise. Such equations arise, for example, when considering mild solutions of stochastic fractional order partial differential equations but also when considering mild solutions of classical stochastic partial differential equations. The key requirement for the equations is a smoothing property of the deterministic evolution operator which is typical in parabolic type problems. We show that if one has access to nonsmooth data estimates for the deterministic error operator together with its derivative of a space discretization procedure,then one obtains error estimates in pathwise H¨older norms with rates that can be read off the deterministic error rates.

References:

[1] K. Fahim, E. Hausenblas and M. Kov´acs, Some approximation results for mild solutions of stochastic fractional order evolution equations driven by Gaussian noise, Stoch PDE: Anal Comp, 2022. https://doi.org/10.1007/s40072-022-00250-0

Gunther Leobacher

University of Graz

Orthogonal projection on manifolds and numerical schemes for SDEs

In [2] and [3], numerical schemes for multidimensional stochastic differential equations (SDEs) with discontinuous drift terms and degenerate diffusions have been studies. The points of discontinuity of the drift coefficient were required to be a smooth (C^4) manifold $\Theta \subseteq \mathbb{R}^d$. On the complement of this manifold, the drift was assumed to be smooth and thus locally Lipschitz continuous.
In [3], a transform was constructed, which transforms the SDE with discontinuous drift into one with Lipschitz continuous drift, thus allowing the application of classical results. This transform requires the so called unique nearest point property of the manifold Θ as well as the function which assigns the nearest point $p(x)$, i.e. the projection, to an ambient point $x \in \mathbb{R}^d \setminus \Theta$. In [1] we study for which classes of manifolds this function p exists and which regularity properties it has. This opens the way for the study of tighter conditions under which the transform method may still be applied.

References:

[1] G. Leobacher and A. Steinicke. Exception sets of intrinsic and piecewise Lipschitz functions. Journal of Geometric Analysis, 32, 2022.

[2] G. Leobacher and M. Szoelgyenyi. A strong order 1/2 method for multidimensional SDEs with discontinuous drift. Ann. Appl. Probab., 27(4):2383–2418, 2017.

[3] G. Leobacher and M. Szoelgyenyi. Convergence of the Euler-Maruyama method for multidimensional SDEs with discontinuous drift and degenerate diffusion coefficient. Numerische Mathematik, 138(1):219–239, 2018.

Felix Lindner

University of Kassel

Strong Convergence Rates for Space-Time Discrete Numerical Approximation Schemes for Stochastic Burgers Equations

The main result presented in this talk establishes strong convergence rates on the whole probability space for explicit space-time discrete numerical approximations for a class of stochastic evolution equations with possibly non-globally monotone coefficients such as stochastic Burgers equations with additive trace-class noise. The key idea in the proof of our main result is (i) to bring the classical Alekseev-Gr¨obner formula from deterministic analysis into play and (ii) to employ uniform exponential moment estimates for the numerical approximations.

Iosif Lytras

University of Edinburgh

New Tamed Langevin MCMC algorithms and their applications

Recently, Langevin-based algorithms have grown in popularity, because of their importance in the fields of sampling and optimization. The majority of works in the current literature deals with problems which involve objective functions with linearly-growing gradients (drift coeeficients in the respective Langevin SDE). Insprired by the taming technology developed in Hutzenthaler et al. (2012) and Sabanis (2013), Sabanis (2016), we propose new Langevin-type MCMC algorithms to deal with cases where the gradient of the objective functions grows superlinearly. We provide nonasymptotic analysis of the new algorithms' convergence properties and their application in optimization.

References:

[1] M. Hutzenthaler, A. Jentzen, and P. E. Kloeden. Strong convergence of an explicit numerical method for sdes with nonglobally lipschitz continuous coefficients. Ann. Appl. Probab., 22(4):1611–1641, 08 2012.

[2] S. Sabanis. A note on tamed euler approximations. Electron. Commun. Probab., 18(47):1–10, 2013.

[3] S. Sabanis. Euler approximations with varying coefficients: the case of superlinearly growing diffusion coefficients. Ann. Appl. Probab., 26(4):2083–2105, 2016.

Andreas Neuenkirch

University of Mannheim

Sharp L^1-Approximation of the log-Heston SDE by Euler-type methods

We study the L^1-approximation of the log-Heston SDE at equidistant time points by Euler-type methods. We establish the convergence order $1/2 - \epsilon$ for $\epsilon > 0$ arbitrarily small, if the Feller index ν of the underlying CIR process satisfies $\nu > 1$. Thus, we recover the standard convergence order of the Euler scheme for SDEs with globally Lipschitz coefficients. Moreover, we discuss the case $\nu \leq 1$ and illustrate our findings by several numerical examples.

Christopher Rauhögger

University of Passau

On the performance of the Euler-Maruyama scheme for multidimensional SDEs with discontinuous drift coefficient

We study the performance of the Euler-Maruyama scheme for systems of SDEs with a piecewise Lipschitz drift coefficient and a Lipschitz diffusion coefficient. We show that an L_p-error rate of at least $1/2-$ is achieved, which generalizes a recent result from [2] for scalar SDEs and improves the known L_2-error rate of at least $1/4$ from [1] for the multi-dimensional setting.

References:

[2] Leobacher, G. and Szoelgyenyi, M. (2018), Convergence of the Euler-Maruyama method for multidimensional SDEs with discontinuous drift and degenerate diffusion coefficient, Numerische Mathematik 138, 219–239.

[2] Mueller-Gronbach, T. and Yaroslavtseva, L. (2020), On the performance of the Euler–Maruyama scheme for SDEs with discontinuous drift coefficient, Ann. Inst. H. Poincar´e Probab. Statist. 56, 1162–1178.

Andreas Rössler

University of Lübeck

A derivative-free Milstein type scheme for SPDEs without commutative noise

We consider the problem of approximating mild solutions of stochastic evolution equations driven by a Q-Wiener process. Therefore, an infinite dimensional version of a derivative-free Milstein type scheme for the time discretization is proposed. For example, the introduced scheme can be applied to a certain class of semilinear stochastic partial differential equations (SPDEs) with commutative as well as non-commutative noise. In case of non-commutative noise, iterated stochastic integrals of the driving Q-Wiener process have to approximated. Finally, the order of convergence and the efficiency of the new scheme will be discussed.

References:

[1] J. Mrongowius, A. Rößler: *On the approximation and simulation of iterated stochastic integrals and the corresponding Levy areas in terms of a multidimensional Brownian motion*, Stoch. Anal. Appl., Vol. 40, No. 3, 397-425 (2022).

[2] C. von Hallern, A. Rößler: *A Derivative-Free Milstein Type Approximation Method for SPDEs covering the Non-Commutative Noise case*, arXiv:2006.08275 (2020).

[3] C. Leonhard, A. Rößler: *Iterated stochastic integrals in infinite dimensions: approximation and error estimates*, Stoch. PDE: Anal. Comp., Vol. 7, No. 2, 209-239 (2019).

Verena Schwarz

University of Klagenfurt

A higher order scheme for jump-diffusion SDEs with discontinuous drift

In this talk we present a strong approximation result for the solution of jump-diffusion stochastic differential equations with discontinuous drift, possibly degenerate diffusion coefficient, and Lipschitz jump-diffusion. We construct a transformation-based jump-adapted quasi-Milstein scheme, which has convergence order $3/4$ under additional piecewise smoothness assumptions to the drift and diffusion coefficient. To obtain this result we show that under slightly stronger assumptions on the coefficients the jump-adapted quasi-Milstein scheme also has convergence order $3/4$.

Michal Sobieraj

AGH University of Science and Technology

On efficient approximation of solutions of SDEs driven by countably dimensional Wiener process and Poisson random measure

In this presentation I refer recent results ([3]) on optimal pointwise approximation of SDEs of the following form

$$\begin{cases} \mathrm{d}X(t) = a(t, X(t))\mathrm{d}t + b(t, X(t))\mathrm{d}W(t) + \int\limits_{\mathcal{E}} c(t, X(t-), y)N(\mathrm{d}y, \mathrm{d}t), \ t \in [0, T], \\ X(0) = \eta, \end{cases} \tag{3}$$

where $T > 0$, $\mathcal{E} = \mathbf{R}^{d'} \setminus \{0\}$, $d' \in \mathbf{N}$, $W = [W_1, W_2, \ldots]^T$ is a countably dimensional Wiener process, and $N(\mathrm{d}y, \mathrm{d}t)$ is a Poisson random measure with an intensity measure $\nu(\mathrm{d}y)\mathrm{d}t$ (see [1], [2]). We assume that $\nu(\mathrm{d}y)$ is a finite Lévy measure on $(\mathcal{E}, \mathcal{B}(\mathcal{E}))$.

In a certain class of coefficients $a : [0, T] \times \mathbf{R}^d \to \mathbf{R}^d$, $b : [0, T] \times \mathbf{R}^d \to l^2(\mathbf{R}^d)$, $c : [0, T] \times \mathbf{R}^d \to \mathbf{R}^d$ we investigate error of a truncated dimension randomized Euler scheme, which uses evaluations of finite number M of components of the Wiener process W. We establish upper bound on its error in the terms of the discretization parameter n and the truncation parameter M. In suitable subclasses we show also corresponding lower bounds on the error of an arbitrary algorithm that is based on (finite dimensional) evaluations of (a, b, c, W).

At the end we present results of numerical experiments performed on GPUs, where we used a suitable implementation (in CUDA C) of the truncated dimension randomized Euler algorithm.

References:

[1] S. N. Cohen, R. J. Elliott. Stochastic Calculus and Applications, 2nd. ed. Springer, 2015.

[2] I. Gyoengy, N. V. Krylov. On stochastic equations with respect to semimartingales I. Stochastics, Volume 4, 1980,1–21.

[3] P. Przybyłowicz, M. Sobieraj, L. Stepien. Efficient approximation of SDEs driven by countably dimensional Wiener process and Poisson random measure, SIAM Journal on Numerical Analysis 60 (2022), 824-855.

Alexander Steinicke

Uuniversity of Leoben

From numerical schemes for SDEs to analysis of Lipschitz maps

Various phenomena in insurance dividend optimization or in modeling the energy market lead to stochastic differential equations (SDEs) with discontinuous drift terms and degenerate diffusion. In real world applications, such drift terms are multidimensional, with discontinuities located at multidimensional manifolds. Numerical schemes and rates for such equations have been given in [3] and [4], requiring the points of discontinuity of the drift coefficient to be a smooth (C^4) manifold Θ. On the complement of this manifold, the drift is assumed to be smooth and thus locally Lipschitz continuous.

In [3], a transform was constructed, which transforms the SDE with discontinuous drift into one with Lipschitz continuous drift, thus allowing the application of classical results.

There, a key lemma was used that allows to conclude Lipschitz continuity of a function from its continuity plus 'intrinsic Lipschitz continuity'. However, the validity of this conclusion relies on the regularity of the manifold. In [2] we coined the notion of a permeable subset of a metric space and showed that permeability is a sufficient condition. In [1] we construct an impermeable Hölder-submanifold of $\mathbb{R}^d$ for which the conclusion fails to hold.

References:

[1] Z. Buczolich, G. Leobacher, and A. Steinicke. Continuous functions with impermeable graphs. arXiv:2201.02159, 2022

[2] G. Leobacher and A. Steinicke. Exception sets of intrinsic and piecewise Lipschitz functions. Journal of Geometric Analysis, 32, 2022.

[3] G. Leobacher and M. Szoelgyenyi. A strong order 1/2 method for multidimensional SDEs with discontinuous drift. Ann. Appl. Probab., 27(4):2383–2418, 2017.

[4] G. Leobacher and M. Szoelgyenyi. Convergence of the Euler-Maruyama method for multidimensional SDEs with discontinuous drift and degenerate diffusion coefficient. Numerische Mathematik, 138(1):219–239, 2018.

Lukasz Stepien

AGH University of Science and Technology in Krakow

Adaptive step-size control for SDEs driven by countably dimensional Wiener process

In this work we deal with global approximation of solutions of scalar stochastic differential equations (SDEs) of the following form

$$\begin{cases} \mathrm{d}X(t) = a(t, X(t))\mathrm{d}t + \sigma(t)\mathrm{d}W(t), \ t \in [0, T], \\ X(0) = \eta, \end{cases}$$

where $T > 0$, η is a suitable random variable, and W is a countably dimensional Wiener process [3,4]. We note that similar, finite-dimensional setting, has been considered e.g. in [1,2]. In particular, infinite-dimensional models enable us to model complicated structure of noise, and are a connection between ordinary and stochastic partial differential equations (SPDEs), while still the mathematical tools for ordinary SDEs can be leveraged.

In [4] we propose a novel, implementable method with adaptive path-independent step-size control and derive its asymptotic rate of convergence. In addition, we show asymptotic lower bounds and asymptotic constants for the main problem under some technical conditions imposed on the equation coefficients. Moreover, the proposed algorithm is asymptotically optimal in the IBC (Information-based complexity) sense. Finally, we support our theoretical findings by numerical results obtained by using the GPU architecture.

The main approximation strategy and upper bound estimations follow the concepts presented in [3] where the authors investigated pointwise approximation problem for SDEs driven by countably dimensional Wiener process and Poisson random measure.

References:

[1] N. Hofmann, T. Mueller-Gronbach, K. Ritter, Optimal approximation of stochastic differential equations by adaptive step-size control, Mathematics of Computation, 69 (2001), 1017–1034.

[2] A. Kałuza, P. Przybyłowicz, Optimal global approximation of jump-diffusion SDEs via path-independent step-size control, Applied Numerical Mathematics, 128 (2018), 24–42.

[3] P. Przybyłowicz, M. Sobieraj, L. Stepien, Efficient approximation of SDEs driven by countably dimensional Wiener process and Poisson random measure, SIAM Journal on Numerical Analysis, 60 (2022), 824–855

[4] L. Stepien, Adaptive step-size control for SDEs driven by countably dimensional Wiener process, in preparation.

Irene Tubikanec

University of Klagenfurt

A splitting method for SDEs with locally Lipschitz drift: Illustration on the FitzHugh-Nagumo model

In this talk, we construct and analyse an explicit numerical splitting method for a class of semi-linear stochastic differential equations (SDEs) with additive noise, where the drift is allowed to grow polynomially and satisfies a global one-sided Lipschitz condition. The method is proved to be mean-square convergent of order 1 and to preserve important structural properties of the SDE. First, it is hypoelliptic in every iteration step. Second, it is geometrically ergodic. Third, it preserves oscillatory dynamics, such as amplitudes, frequencies and phases of oscillations, even for large time steps. Our results are illustrated on the stochastic FitzHugh-Nagumo model and compared with known mean-square convergent tamed/truncated variants of the Euler-Maruyama method.

References:

[1] E. Buckwar, A. Samson, M. Tamborrino, I. Tubikanec. A splitting method for SDEs with locally Lipschitz drift: Illustration on the FitzHugh-Nagumo model, Applied Numerical Mathematics, 179 (2022), 191–220. https://doi.org/10.1016/j.apnum.2022.04.018

Tomasz Tyranovski

Max Planck Institute for Plasma Physics

Data-driven structure-preserving model reduction for stochastic Hamiltonian systems

In this work we demonstrate that SVD-based model reduction techniques known for ordinary differential equations, such as the proper orthogonal decomposition, can be extended to stochastic differential equations in order to reduce the computational cost arising from both the high dimension of the considered stochastic system and the large number of independent Monte Carlo runs. We also extend the proper symplectic decomposition method to stochastic Hamiltonian systems, both with and without external forcing, and argue that preserving the underlying symplectic or variational structures results in more accurate and stable solutions that conserve energy better than when the non-geometric approach is used. We validate our proposed techniques with numerical experiments for a semi-discretization of the stochastic nonlinear Schrödinger equation and the Kubo oscillator.

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The BDF2-Maruyama Method for the Stochastic Allen–Cahn Equation with Multiplicative Noise

We investigate the numerical approximation of the stochastic Allen–Cahn equation on a bounded domain $\mathcal{D}$ under Dirichlet boundary conditions and with multiplicative noise. The considered numerical method combines the two-step backward differentiation formula (BDF2) for the temporal discretization in conjunction with an abstract Galerkin scheme for the spatial approximation. In dependence on the regularity of the exact solution we derive a rate of convergence for the BDF2-Maruyama method with respect to the root-mean-square error in discrete analogues of the spaces $L^\infty([0,T];L^2(\mathcal{D}))$ and $L^2([0,T];H_0^1(\mathcal{D}))$. Our error analysis is based on the variational approach for stochastic evolution equations. Finally, several numerical experiments illustrate our theoretical results, where a finite element method is used as an example for a Galerkin scheme.