# Matrix valued stochastic processes built upon Wishart, Gamma, and Laplace distributions

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### Abstract

There are many methods to build a matrix valued stochastic process. This work present a method that is based on matrix variate gamma distributions with the shape parameter taking on any positive value. The extension includes singular Wishart as well as (nonsingular) matrix variate gamma distributions as special cases. The group properties of this class provide a natural alternative to the classical convolution properties that are crucial in the study of infinite divisibility. Our results provide further clarification regarding the lack of infinite divisibility of Wishart distributions, a classical observation of Paul Lévy, [1]. In particular, we clarify why the row/column vectors in the off-diagonal blocks are infinitely divisible. A new class of matrix variate Laplace distributions arises naturally in this set-up as the distributions of the off-diagonal blocks of random gamma matrices. Along the way, we obtain several convenient stochastic representations of the discussed classes of matrix distributions. We show that the matrix gamma distributions is symmetrization of the triangular matrix Rayleigh distribution, which proposed as a natural extension of the classical Rayleigh distribution. Finally, a connection of the extended matrix Gamma distributions to matrix valued Lévy processes with a vector argument is made. Namely, a Lévy process called a matrix gamma-Laplace motion is obtained by subordination of the triangular Brownian motion of a vector argument to the vector valued gamma motion of a vector argument. This process serves to define the triangular matrix valued Rayleigh process which, through its symmetrization, leads to a new process, named the matrix variate gamma process. This process when taken at a properly set one dimensional argument has the matrix valued gamma distributions as its marginals and is an alternative to the Lévy processes studied in [2]. The presented results are obtained jointly with Tomasz J. Kozubowski and Stepan Mazur.

### **Keywords**

Infinite divisibility, singular Wishart distributions, Laplace distribution, Lévy motion.

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## References

- LÉVY, P. The arithmetical character of the wishart distribution. Mathematical Proceedings of the Cambridge Philosophical Society 44 (1948), 295–297.
- [2] PÉREZ-ABREU, V., AND STELZER, R. Infinitely divisible multivariate and matrix gamma distributions. *Journal of Multivariate Analysis 130* (2014), 155–175.

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