Alessio Pomponio

The Chern-Simons-Schrödinger system: static and non-static solutions

Dipartimento di Meccanica, Matematica e Management
Politecnico di Bari, Italy

Abstract

In this talk we deal with the Chern-Simons-Schrödinger system

$$\begin{cases} iD_0\phi + (D_1D_1 + D_2D_2)\phi + |\phi|^{p-1}\phi = 0, \\ \partial_0 A_1 - \partial_1 A_0 = \mathrm{Im}(\bar{\phi} D_2 \phi), \\ \partial_0 A_2 - \partial_2 A_0 = -\mathrm{Im}(\bar{\phi} D_1 \phi), \\ \partial_1 A_2 - \partial_2 A_1 = \frac{1}{2}|\phi|^2, \end{cases} \tag{$\mathcal{P}$}$$

where the unknowns are (ϕ, A_0, A_1, A_2) and $(t, x) \in \mathbb{R} \times \mathbb{R}^2$.

If we seek standing wave solutions of the form

$$\begin{aligned} \phi(t,x) &= u(|x|)e^{i\omega t}, & A_0(x) &= k(|x|), \\ A_1(x) &= -\frac{x_2}{|x|^2} h(|x|), & A_2(x) &= \frac{x_1}{|x|^2} h(|x|), \end{aligned}$$

with $\omega \geqslant 0$, as a first step we evaluate all the components of the gauge potential, A_i with $i = 0, 1, 2$, in terms of u and then we study the following nonlocal planar equation

$$-\Delta u + \left(\omega + \frac{h_u^2(|x|)}{|x|^2} + \int_{|x|}^{+\infty} \frac{h_u(s)}{s} u^2(s)\, ds \right) u = |u|^{p-1} u,$$

where

$$h_u(r) = \int_0^r \frac{s}{2} u^2(s)\, ds.$$

We present some existence results for the Chern-Simons-Schrödinger system ($\mathcal{P}$). In particular we find non-static solutions, in the case $\omega > 0$, and static solutions under a large-distance fall-off requirement on the gauge potentials, whenever $\omega = 0$.