Existence results for nonlinear Choquard equations with potentials

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In this talk we will address the problem of finding positive solutions to Choquard equations in the presence of a potential $V(x)$

$$\begin{cases} -\Delta u + V(x)u = (I_\alpha * |u|^p)\,|u|^{p-2}\,u, \quad x \in \mathbb{R}^N, \\ u \in H^1(\mathbb{R}^N), \end{cases}$$

where $N \geq 2$, $\alpha \in (0, N)$; I_α represents the Riesz operator of order α, defined for each point $x \in \mathbb{R}^N \setminus \{0\}$ by

$$I_\alpha(x) = \frac{A_\alpha}{|x|^{N-\alpha}}, \quad \text{where} \quad A_\alpha = \frac{\Gamma(\frac{N-\alpha}{2})}{\Gamma(\alpha/2)2^\alpha \pi^{N/2}},$$

and Γ is the Gamma function; the exponent p satisfies

$$\frac{N-2}{N+\alpha} < \frac{1}{p} < \frac{N}{N+\alpha}.$$

The solutions we will find will not be least action ones, so that particular attention will be devoted to the problem of recovering compactness properties. Of special interest will be the case $p < 2$, as in this situation we will face the non-local feature of the problem. The presented results are contained in the papers [1, 2].

References

[1] L. Maia, B. Pellacci, D. Schiera, *Positive bound states to the nonlinear Choquard equations in the presence of nonsymmetric potentials,* Minimax theory and applications (2) **7** (2022).

[2] L. Maia, B. Pellacci, D. Schiera, *Symmetric positive solutions to nonlinear Choquard equations with potentials.* Calc. Var. Partial Differential Equations (2) **61** (2022).