SYMMETRY RESULTS FOR COMPACTLY SUPPORTED SOLUTIONS OF THE 2D STEADY EULER EQUATIONS

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ABSTRACT. In this paper we prove symmetry of compactly supported solutions of the 2D steady Euler equations. Assuming that $\Omega = \{x \in \mathbb{R}^2 : \ u(x) \neq 0\}$ is an annular domain, we prove that the streamlines of the flow are circular. We are also able to remove the topological condition on Ω if we impose regularity and nondegeneracy assumptions on u at $\partial\Omega$. The proof uses that the corresponding stream function solves an elliptic semilinear problem $-\Delta\phi = f(\phi)$ with $\nabla\phi = 0$ at the boundary. One of the main difficulties in our study is that f is not Lipschitz continuous near the boundary values. However, $f(\phi)$ vanishes at the boundary values and then we can apply a local symmetry result of F. Brock to conclude.

In the case $\partial_\nu u \neq 0$ at $\partial\Omega$ this argument is not possible. In this case we are able to use the moving plane scheme to show symmetry, despite the possible lack of regularity of f. We think that such result is interesting in its own right and will be stated and proved also for higher dimensions. The proof requires the study of maximum principles, Hopf lemma and Serrin corner lemma for elliptic linear operators with singular coefficients.