Overdetermined problems and shape optimization in cones

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We present some results about the problem of characterizing domains in cones for which a solution of a partially overdetermined problem exists. This is related to the question of characterizing constant mean curvature surfaces with boundary in cones. In recent papers in collaboration with G. Tralli it is proved that if the cone is convex the only domains or surfaces with the above properties are spherical sectors or spherical caps. This should still hold if the cone is almost convex but it is not expected to be true for general nonconvex cones.

Some recent results in collaboration with A. Iacopetti and T. Weth show that these questions are related to the study of the first nontrivial Neumann eigenvalue of the Laplace-Beltrami operator on domains on the unit sphere. In particular, this allows to determine classes of nonconvex cones for which the spherical sectors are not the minimizers for the associated shape optimization problem for the torsional energy functional. Finally, by a concentration-compactness argument, it is proved that minimizers for the shape optimization problem do exist in some cases but are not radial domains. A similar break of symmetry result holds for constant mean curvature surfaces in some nonconvex cones.