Biharmonic nonlinear scalar field equations

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Abstract

Let us consider the biharmonic nonlinear equation

$$\Delta^2 u = g(x,u) \qquad \text{in } \mathbb{R}^N$$

with a Carathéodory function $g : \mathbb{R}^N \times \mathbb{R} \to \mathbb{R}$, $N \geq 5$. We prove a Brezis-Kato-type regularity result for weak solutions of the above equation. Then, we show the existence of ground state solutions provided that g has a general subcritical growth at infinity. We also obtain a new biharmonic logarithmic Sobolev inequality

$$\int_{\mathbb{R}^N} |u|^2 \log |u| \, dx \leq \frac{N}{8} \log\Big(C \int_{\mathbb{R}^N} |\Delta u|^2 \, dx \Big),$$

for $u \in H^2(\mathbb{R}^N)$ with $\int_{\mathbb{R}^N} u^2 \, dx = 1$ and $0 < C < \big(\frac{2}{\pi e N}\big)^2$.