

Singular quasilinear critical Schrödinger equations in \mathbb{R}^N

Laura Baldelli
Institute of Mathematics, Polish Academy of Sciences
lbaldelli@impan.pl

Abstract

In this talk we discuss some recent results concerning singular quasilinear Schrödinger problems in the entire \mathbb{R}^N of the type

$$-\Delta_p u - \frac{\alpha}{2} \Delta_p(|u|^\alpha)|u|^{\alpha-2}u = \lambda V(x)|u|^{k-2}u + \beta K(x)|u|^{p^*-2}u \quad \text{in } \mathbb{R}^N, \quad (1)$$

where $1 < p < N$, $1 < k < p^*$, $0 < \alpha < 1$. Moreover, in problem (1) appears a nonlinearity with both a critical term, in the sense of the critical Sobolev's exponent $p^* = Np/(N-p)$, and a subcritical term, both involving nonnegative nontrivial weights V , K and positive real parameters λ, β . Quasilinear equations of the type (1) are related to the existence of standing wave solutions for the following Schrödinger equation

$$i\partial_t \psi = -\Delta \psi + W(x)\psi - \varphi(|\psi|^2)\psi - \kappa \Delta \varrho(|\psi|^2) \varrho'(|\psi|^2)\psi,$$

where $\psi : \mathbb{R} \times \mathbb{R}^N \rightarrow \mathbb{C}$, W is a given potential, κ is a real constant and φ, ϱ real functions of essentially pure power form, and cover several physical models coming from plasma physics as well as high-power ultra short laser in matter, fluid mechanics, the theory of Heisenberg ferromagnets and magnons. In particular, under suitable conditions on the exponent k in the nonlinearity, we obtain multiplicity results with negative and positive energy depending on the range of the parameters λ, β , overcoming the double loss of compactness due both to the critical Sobolev's exponent p^* and to the unboundedness of the domain. We analyze also the case of nonnegative nontrivial weights satisfying some symmetry conditions with respect to a certain group $T \subset O(N)$, where $O(N)$ is the group of orthogonal linear transformations in \mathbb{R}^N . Our proofs rely on variational tools, including concentration compactness principles because of the delicate situation of the double lack of compactness. In addition, a necessary reformulation of the original problem in a suitable variational setting produces a singular function, delicate to be managed.