Topics in variational problems arising from models in physics (24–29/07/22, Będlewo)

A powerful construction beyond powers: multiplicity of solutions for a Hartree-type equation

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In the research for multiple solutions to equations of the type

$$-\Delta u + \mu u = \left(\frac{1}{|x|^{N-\alpha}} * F(u)\right) f(u) \quad \text{on } \mathbb{R}^N$$
(1)

a standard way to reach the goal is to assume f = F' as a *power*, i.e. $f(u) = |u|^{p-1}u$ [Lions, 1980]. What happens when we drop this homogeneity? This question was answered by [Berestycki, Lions, 1983] in the case of *local* nonlinearities: but actually, their proof heavily relies on the locality of the source.

Aim of this talk is to reach the same goal, but in the case of general *nonlocal* nonlinearities of convolution type [Moroz, Van Schaftingen, 2015], i.e. dealing with equation (1). The construction will be based on a minimax approach and on the introduction of suitable *multidimensional annuli*, and it will deal also with the (less-studied) case of *even* nonlinearities f.

As a further application of the previous construction, in the second part of the talk we will treat the case of *normalized* solutions, that is when the L^2 -mass is prescribed in advance.

The presented results are in collaboration with Silvia Cingolani and Kazunaga Tanaka.