On the number of critical points of PDE solutions

The calculation of the number of critical points of a solution of a PDE is an old and classic problem. Some powerful techniques (Morse theory, topological degree) allow to give estimates on the total number of critical points. However, the exact calculation requires additional ideas. In the famous Gidas-Ni-Nirenberg Theorem the uniqueness of the critical point of the solution to

$$\begin{cases} -\Delta u = f(u) & in\ \Omega \\ u > 0 & in\ \Omega \\ u = 0 & on\ \partial\Omega \end{cases} \tag{0.1}$$

in convex and symmetric domains was proved.

In this lecture we consider solutions to (0.1) in which the symmetry of Ω is lacking. In particular we will examine the case of the domain with a small hole and we show, in suitable situation, the existence of exactly *two* critical points. An important remark is that the number of the critical points is influenced by the location of the hole.

Some interesting cases include the torsion problem $f \equiv 1$ and the Robin function.

The results are obtained in collaboration with F. De Regibus, F. Gladiali, P. Luo and S. Yan.