New Gagliardo-Nirenberg inequalities and applications to biharmonic NLS

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We present new existence results for nontrivial standing wave solutions of the biharmonic Nonlinear Schrödinger equation of mixed dispersion type

$$i\partial_t\psi - \Delta^2\psi + 2\Delta\psi = |\psi|^{p-2}\psi, \qquad \psi(0,x) = \psi_0(x), \quad (t,x) \in \mathbb{R}\times\mathbb{R}^d.$$

To deduce the existence of orbitally stable standing waves with a given mass as in Cazenave-Lions ('82), we prove a new class of Gagliardo-Nirenberg inequalities involving the Helmholtz operator instead of the Laplacian. Having explained the relevance of such inequalities for our analysis, we comment on their proofs and related questions from Harmonic Analysis. Finally, we present a related symmetry-breaking phenomenon for small masses that was recently discovered by Lenzmann and Weth.

This is joint work with A. Fernández, L. Jeanjean, M. Mariş.