A Quasilinear Wave Equation Based on a Kerr-Nonlinear Maxwell Model: Well-Posedness and Time-Periodic Solutions

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We consider the (1+1)-dimensional quasilinear wave equation $g(x)w_{tt} - w_{xx} + h(x)(w^3)_{tt} = 0$ on $\mathbb{R}\times\mathbb{R}$ which arises in the study of localized electromagnetic waves modeled by Kerr-nonlinear Maxwell equations. The particular feature of the model is that $h(x) = \gamma\delta_0(x)$ is a multiple of a delta-distribution supported at 0 and g is a bounded potential. We first consider well-posedness of the initial value problem. Then we prove by variational methods the existence of infinitely many standing, time-periodic, real-valued, spatially expontially localized waves. Theses waves are necessarily polychromatic, i.e., infinitely many Fourier-modes in time are occupied. An extension to the case where $h(x)$ is not a distribution but a bounded function will be indicated.

References

[1] S. Kohler and W. Reichel, Breather solutions for a quasilinear $(1+1)$-dimensional wave equation, *Stud. Appl. Math.*, 2022; 148: 689– 714.

[2] S. Ohrem, W. Reichel, and R. Schnaubelt, Well-posedness for a (1+1)-dimensional wave equation with quasilinear boundary condition, in preparation.