60LB Geometry and Singularities
60th anniversary of Lev Birbrair

July 17-21, 2023

Booklet of abstracts

Banach Center
Będlewo, Poland

Jagiellonian University, 2023

About the conference

Conference website:

https://www.impan.pl/en/activities/banach-center/conferences/23-birbair60

Hosting institution:

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Foreword

The present conference is organised in order to celebrate the *60th anniversary* of **Lev Birbrair**.

Lev Birbrair was one of the first mathematicians to work on the classification of semi-algebraic sets up to *bi-Lipschitz equivalence*. He pointed out three natural approaches of Lipschitz classification: with respect to the inner (geodesic) metric, the outer metric, and ambient Lipschitz classification. His first and very important contribution is the theory of *Hölder complexes* giving a complete Lipschitz classification of semi-algebraic surface germs with respect to the inner metric.

Together with Tadeusz Mostowski he introduced the notion of *Lipschitz Normal Embedding* which became very important in recent research.

Together with Alexandre Fernandes he discovered two very interesting phenomena: *fast loops and separating sets* whose existence is a serious obstruction for metrical conicalness of complex surface germs. Using the fast loop technique Lev Birbrair, Alexandre Fernandes and Walter Neumann showed that the germs of surfaces can be ambient topological equivalent, but not inner bi-Lipschitz equivalent.

Next, Lev Birbrair, Walter Neumann and Anne Pichon obtained a complete description of the inner Lipschitz geometry of germs of normal surfaces, called a *refined thick and thin decomposition* which is a complete bi-Lipschitz invariant with respect to the inner metric.

The most recent contribution of Lev Birbrair in Lipschitz Geometry is the creation of *Metric Knot Theory* together with Andrei Gabrielov, a very promising subject.

We wish you all a nice and fruitful stay in Będlewo!

The organizers

Contents

Abstracts of talks

The tangent cone of the medial axis

July 18
16:55

Adam Białożyt
Jagiellonian University

The medial axis M_X of a given closed, nonempty, proper subset $X \subset \mathbb{R}^n$ is the set of those points x in the ambient space that have more than one closes point in X w.r.t. the Euclidean distance $d(x, X)$. Only recently, starting from the works of L. Bibrair, M. Denkowski, D. Siersma, a relation between M_X and the singularities of X has been noted.

This talk is twofold, both its parts being connected by the tangent cone of the medial axis of a given definable set.

First, we present a recent extension to complete Riemannian manifolds of a surprising stability result of the medial axis in continuous families obtained together with A. Denkowska and M. Denkowski. This 'inner semi-continuity' yields a useful tangent cone criterion for the reaching of singularities of X by M_X in the definable case.

We apply next this stability result (in the Eculidean space) establishing a relation between the tangent cone of M_X at a given point a and the medial axis of the set of points $m(a) \subset X$ realising the distance $d(a, X)$. As a consequence, a lower bound for the dimension of the medial axis of X in terms of the dimension of the medial axis of $m(a)$ is obtained. This appears to be the missing link to the full description of the medial axis' dimension.

Concordance group of knots and stable commutator length in braid groups

July 18
14:45

Michael Brandenbursky
Ben Gurion University

In this talk I will describe an interesting connection between 4d topology and geometric group theory. More precisely, I will define quasi-homomorphisms from braid groups to the concordance group of knots and examine its properties and consequences of its existence. In particular, I will provide a relation between the stable four ball genus in the concordance group and the stable commutator length in braid groups, and produce examples of infinite families of concordance classes of knots with uniformly bounded four ball genus. I will also provide applications to the

geometry of the infinite braid group. In particular, I will show that its commutator subgroup admits a stably unbounded conjugation invariant norm. If time permits, I will describe an interesting connection between the concordance group of knots and number theory. This work is partially joint with Jarek Kedra.

Poincaré-Hopf Theorem "à la mode Lipschitz"

July 21
16:00

Jean-Paul Brasselet
CNRS

This is a joint work with Tadeusz Mostowski and Thuy Nguyen Thi Bich.

For a long time it was thought that the Poincaré-Hopf theorem was false in the case of singular varieties. There are indeed counter-examples, including considering stratified vector fields on a singular manifold endowed with a Whitney stratification.

It was Marie-Hélène Schwartz who, in 1965, showed that the theorem is true by considering "radial" vector fields. Unfortunately, her construction, if it uses the nice idea of radial fields, is done within the framework of Whitney stratifications and the implementation is very delicate and technical.

With Tadeusz Mostowski and Thuy Nguyen Thi Bich, we show that, within the framework of Lipschitz stratifications, the implementation of the idea of M.-H. Schwartz is simpler and prettier.

Main contributions of Lev Birbrair to Lipschitz Geometry, Part I

July 17
10:00

Alexandre Fernandes
Federal University of Ceará

In this talk, divided into two parts, we plan to present the main results, concepts and tools introduced by Lev Birbrair (alone and with collaborators) in the study of Lipschitz Geometry of Singularities. We plan to address the influence of Lev Birbrair in the development of the following topics:
Part I: Metric homology; Fast loops; Separating sets; Local inner Lipschitz geometry of complex surfaces; Outer Lipschitz geometry of real surfaces (pizza and beyond).

Equimultiplicity of families with constant Milnor number

July 19
11:30

Javier Fernández de Bobadilla
BCAM

I will summarize the main steps of our proof of the isilates singularity family case of Zariski's multiolicity conjecture. It is based on the construction of symplectic representatives of the monodromy with special dynnamics, and a Floer theoretic computation.
Joint with T. Pełka

Pairs of Lipschitz Normally Embedded Hölder Triangles

July 18
10:00

Andrei Gabrielov
Purdue University

The topic of this talk is part of the project *Outer Lipschitz classification of definable surface germs* initiated by Lev Birbrair. A Lipschitz normally embedded (LNE) Hölder triangle is a building block of the Lipschitz geometry of surface germs. Two such triangles are bi-Lipschitz equivalent when their exponents are equal. However, outer Lipschitz geometry of a pair of LNE Hölder triangles is rather non-trivial. We define a complete invariant, called $\sigma\tau$-invariant, of a "normal" pair of Hölder triangles: Two such pairs are outer bi-Lipschitz equivalent if, and only if, their $\sigma\tau$-invariants are the same.

On certain type of singular varieties with smooth subvarieties

July 21
16:55

Maria del Rosario Gonzalez Dorrego
Madrid Autonomous University

Let k an algebraically closed field. Let $char k = 0$. We study multiplicity-r structures on varieties $r \in \mathbb{N}$. Let Z be a reduced irreducible nonsingular $(n-1)$-dimensional variety such that $rZ = X \cap F$, where F is a $(N-1)$-fold in $\mathbb{P}^N$, X is a normal n-fold with certain type of singularities. We study when $Z \cap Sing(X) \neq \emptyset$.

The multiplicity mod 2 is not a bi-lipschitz invariant

July 21
10:00

Zbigniew Jelonek
IMPAN

We give examples of two four dimensional real algebraic cones in $\mathbb{R}^8$ which have degrees different mod 2 but which are eqiuivalent by semi-algebraic bi-lipschitz homeomorphisms.

This is a joint work with Alex Fernandes and Edson Sampaio.

Characteristic-free approach to unfoldings (and other results)

July 20
16:00

Dmitry Kerner

Ben Gurion University

In 40's Whitney studied maps of $\mathcal{C}^\infty$ manifolds. When a map is not an immersion/submersion, one tries to deform it locally, in hope to make it 'generic'. This approach has led to the rich theory of stable maps, developed by Thom, Mather and many others. The main 'engine' was vector field integration. This chained the whole theory to the $\mathcal{C}^\infty$, or R/C-analytic setting.

I will present a purely algebraic approach, studying maps of germs of Noetherian schemes, in any characteristic. The relevant groups of equivalence admit 'good' tangent spaces. One has the theory of unfoldings (triviality and versality). Then I will discuss the new results on stable maps and theorems of Mather-Yau/Gaffney-Hauser.

Extensions of regular maps in real algebraic geometry

July 19
10:00

Wojciech Kucharz

Jagiellonian University

Let X, Y be affine real algebraic varieties and let A be a Zariski closed subvariety of X. It is well known that every real-valued regular function on A has a regular extension to X. However, this does not carry over to Y-valued regular maps on A, that is, it can happen that a regular map from A to Y is not the restriction of any regular map from X to Y. We prove that if X is compact in the Euclidean topology and Y is a rational variety which is homogeneous for some linear algebraic group, then every null homotopic map from A to Y is the restriction of a regular map from X to Y. This result is not at all obvious even if X and Y are unit spheres.

A Bochnak-Siciak theorem for Nash functions over real closed fields

July 21
11:30

Krzysztof Kurdyka

University Savoie Mt Blanc

Let R be a real closed field. We prove that if R is uncountable, then a function $f\colon U \to R$ defined on an open semialgebraic set U in R^n, with $n \geq 2$, is a Nash function whenever for every affine 2-plane Q in R^n the restriction $f|_{U\cap Q}$ is a Nash function (some condition on the shape of U is required if R is not Archimedean). This is an analog of the Bochnak–Siciak theorem established in the real analytic setting. We also provide an example showing that uncountability of R is essential. Joint work with W. Kucharz.

Ambient Lipschitz Geometry of Lipschitz normally embedded surfaces

July 21
14:45
online

Davi Lopes Alves de Medeiros
Federal University of Ceará

The aim of the talk is to present an ambient Lipschitz complete classification result for semi-algebraic surface germs in $\mathbb{R}^3$ with simple link i.e. homeomorphic either to a segment or a circle.

Multi $\mathcal{K}$-Lipschitz equivalence and geometry of algebraically parametrized surfaces

July 20
16:55

Rodrigo Mendes Pereira
International Integration University of Afro-Brazilian Lusophony

This lecture will present recent progress on the outer geometry of surfaces in Euclidean space that are locally parametrized by germs of analytic/polynomial maps $F\colon (\mathbb{R}^2, p) \to (\mathbb{R}^n, F(p))$. Our investigation is located via natural decomposition of the rank of the Jacobian matrix ($\text{rank}(DF(p) \in \{1, 0\}$) and the study of polar zones nearby of its tangent cone (that can be parametrized in many cases). We notice that the behaviour of a parametrized surface nearby of its polar zones is connected with the asymptotic behaviour of its coordinates functions, i.e., its Multi-$\mathcal{K}$-Lipschitz equivalence class (this topic for surfaces is recently investigated in `https://arxiv.org/abs/2304.06610`) jointly with Lev Birbrair. We discuss this connection, presenting some flatness Lipschitz criterion and distinction from that. If time permits, we also discuss about the special case $F\colon (\mathbb{R}^2, p) \to (\mathbb{R}^4, f(p))$ that is related with knot theory.

The labeled dual tree of a fold map germ from $\mathbb{R}^3$ to $\mathbb{R}^5$

July 20
14:45

Juan Antonio Moya Pérez
València University

Let $f : (\mathbb{R}^3, 0) \to (\mathbb{R}^5, 0)$ be an analytic map germ with isolated instability. Its link is a stable map which is obtained by taking the intersection of the image of f with a small enough sphere S^4_ε centered at the origin in $\mathbb{R}^5$. If f is of fold type, we define a labeled tree associated to its link and prove that is a complete topological invariant for it. As an application we obtain all the topological classes contained in the $\mathcal{A}^2$-class $(x, y, z^2, xz, 0)$, proving that in this case the number of branches of the double point curve $D(f)$ is a complete topological invariant. Joint work with J. J. Nuño-Ballesteros.

The Bruce-Roberts number of a function on an ICIS

July 20
11:30

Juan José Nuño-Ballesteros
València University

Given a germ of analytic variety $(X,0)$ in $(\mathbb{C}^n,0)$, the Bruce-Roberts number of a function $f:(\mathbb{C}^n,0)\to\mathbb{C}$ is an invariant which generalises the Milnor number. We will present the basic properties of this invariant as well as the relative version when we consider the restriction of the function to the variety $(X,0)$. When $(X,0)$ is a hypersurface with isolated singularity (IHS) or more generally, when $(X,0)$ is a complete intersection with isolated singularity (ICIS) we find formulas which allow to compute the Bruce-Roberts numbers in a simpler way, in terms of other well known invariants. Finally, we will see the relationship between the Bruce-Roberts number and the logarithmic characteristic variety of $(X,0)$.

Augmentation of singularities: μ/τ-type conjectures and simplicity

July 17
14:45

Raúl Oset Sinha
València University

For function germs $g:(\mathbb{C}^n,0)\to(\mathbb{C},0)$ it is well known that $1\leq\frac{\mu(g)}{\tau(g)}$ and it has recently been proved by Liu that $\frac{\mu(g)}{\tau(g)}\leq n$. We give an upper bound for the codimension of map-germs $f:(\mathbb{C}^n,0)\to(\mathbb{C}^p,0)$ given as augmentations of other map-germs with which we prove the analog to the first inequality (known as Mond's conjecture) for augmentations $h:(\mathbb{C}^n,0)\to(\mathbb{C}^{n+1},0)$. Furthermore, we show that the quotient given by the image Milnor number and the codimension of any augmentation in the pair of dimensions $(n,n+1)$ is less than $\frac{1}{4}(n+1)^2$ and prove the analog to the second inequality for map-germs with $n=1$ and augmentations with $n=2,3$. We then prove a characterization of when a map-germ is an augmentation, finding a counterexample for the characterization given by Houston. Next, we give sufficient conditions for when the augmentation is independent of the choice of stable unfolding by studying different notions of equivalence of unfoldings. Moreover, these results allow us to give sufficient conditions for the simplicity of an augmentation, providing context to locate the moduli for non-simple augmentations.

Perturbation of polynomials and linear operators

July 18
11:30

Adam Parusiński
Côte d'Azur University

Theory of perturbation of polynomials and linear operators is motivated mainly by partial differential equations. In this talk I give an account of some recent results. In particular, with A. Rainer, we show that the roots of monic polynomials of one complex variable depending smoothly on a single real parameter t are locally absolutely continuous with respect to t, and we give an optimal estimate of their Sobolev regularity.

In another paper with Guillaume Rond, we show that the analytic family of normal matrices depending on a multiparameter can be locally analytically diagonalized provided the discriminant of its characteristic polynomial is normal crossings, thus improving the Kurdyka-Paunescu result. A similar statement holds for the singular value decomposition of families of arbitrary matrices.

Moderately Discontinuous Homology of surface singularities

July 20
10:00

Maria Pe Pereira
Madrid Complutense University

We see how the Moderately Discontinous Homology can be used to check non metric conicalness for surfaces with the inner metric.

Complete outer Lipschitz classification of complex surface singularities

July 17
16:00
online

Anne Pichon
Aix-Marseille University

I will revisit the complete inner Lipschitz classification of complex surface germs obtained 10 years ago with Lev and Walter Neumann and then, present the complete classification of complex surfaces germs up to outer Lipschitz homeomorphism. The new tools involve non archimedean geometry, in particular the non-archimedean link which is a generalization of the valuative tree introduced by Favre and Jonsson, and ultrametrics on what we call the logarithmic link of the singularity. This is a joint work with Lorenzo Fantini and Walter Neumann.

On semialgebraic $\mathcal{C}^p$ category

July 20
17:45

Rafał Pierzchała
Jagiellonian University

I will give some conditions which guarantee that a given semialgebraic function (on an open set in $\mathbb{R}^N$) is of class $\mathcal{C}^p$ or of class $\mathcal{C}^p$ with locally Lipschitz partial derivatives of order p.

Main contributions of Lev Birbrair to Lipschitz Geometry, Part II

July 17
11:30

José Edson Sampaio
Federal University of Ceará

In this talk, divided into two parts, we plan to present the main results, concepts and tools introduced by Lev Birbrair (alone and with collaborators) in the study of Lipschitz Geometry of Singularities. We plan to address the influence of Lev Birbrair in the development of the following topics:
Part II. Local inner Lipschitz geometry of real surfaces; Lipschitz regularity; Invariance of the multiplicity; Metric knots and Lipschitz embeddings.

Singularity Theory on Spider Spaces

July 18
16:00

Dirk Siersma
Utrecht University

Spiders are models for graphs, consiting of a central body (the 'head'), wicht is conected by a number of legs to certain fixed points in a metric space. Each leg consists of a number of connected bars, which fixed length, which are allow the move. These type of legs are also known as 'robot arms'. Spiders are examples of arachnoid mechanisms. Planar spiders are those, who can move in the affine plane.

In this talk we will first describe spider space (i.e the configuration space of spiders) and discuss its singularities. After that we will consider a Hooke Energy function and describe is critical points and discuss the Morse-Bott, including Morse indices. There is direct relation with the critical points of a squared distance function. Details will appear in joint work with with Marcej Denkowski and Gaiane Panina.

On the Casas–Alvero conjecture

July 17
16:55

Mark Spivakovsky
Paul Sabatier Toulouse University

Let k be a field, d a strictly positive integer and x an independent variable. Let $f \in k[x]$ be a monic polynomial of degree d. For $i \in \{1, \dots, d-1\}$, let $f^{(i)}$ be the i-th derivative of f (the i-th Hasse derivative in case char $k > 0$). Assume that for all $i \in \{1, \dots, d-1\}$ the polynomial $f^{(i)}$ has a common factor with f. The Casas–Alvero conjecture asserts that under these hypotheses there exists $a \in k$ such that $f(x) = (x-a)^d$.

If $\mathrm{char}(K) = p > 0$, the Conjecture is false in general.

Let us write $\mathrm{CA}_{d,p}$ for the statement "The Casas-Alvero conjecture holds for polynomials of degree d over fields of characteristic p".

The following equivalences are known for each d:

$\mathrm{CA}_{d,0}$ holds $\iff$ $\mathrm{CA}_{d,p}$ holds for some prime number p $\iff$ $\mathrm{CA}_{d,p}$ holds for all but finitely many primes p.

A prime number p is said to be a **bad prime** for d if $\mathrm{CA}_{d,p}$ is false.

In this talk we will discuss an approach to the conjecture that consists in first proving it for some small degree d, compiling lists of bad and good primes for that d and then deducing the conjecture for all the degrees of the form dp^ℓ, where ℓ is a positive integer and d a good prime for d. At the end of the talk we will discuss a recent result (joint with D. Schaub) that gives a long (but not exhaustive) list of good primes for each d.
