Title: Collapsed Anosov flows
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Abstract:
Partially hyperbolic systems arise naturally when studying robust dynamical behavior such as robust transitivity and stable ergodicity. It is also a natural setting to understand statistical properties of dynamical systems beyond uniform hyperbolicity. In recent years, many fine dynamical and statistical properties have been obtained for certain classes of partially hyperbolic diffeomorphisms (such as derived from Anosov, skew-products, or perturbations of time one maps of Anosov flows). On the classification side, much effort was made to show that, at least under some topological restrictions, these settings encompassed all partially hyperbolic diffeomorphisms in dimension 3.

Recent examples have shown that there are many more types of partially hyperbolic diffeomorphisms, with still mysterious dynamical properties. Motivated by these examples, as well as work on the classification problem, we introduced a new class of partially hyperbolic diffeomorphisms in 3-manifolds, that we called *collapsed Anosov flows*.

In this mini-course we will present this class and discuss some of their attributes. Our aim is to advertise the fact that this class can both represent a goal for the classification of partially hyperbolic diffeomorphisms in dimension 3, as well as a natural category for the study of some finer dynamical properties.

In particular, we aim to explain that: The class of collapsed Anosov flows is "large" in many sense; One can prove accessibility (and ergodicity) for partially hyperbolic diffeomorphisms in that class; And, finally, via the link with self orbit equivalences of Anosov flow, it gives us a map for where to look for "undiscovered" collapsed Anosov flows.

The mini-course will be split into four lectures, which will roughly cover the following topics:

- In lecture 1, we will cover some basic definitions, define collapsed Anosov flows, as well as stronger variants of this class, give examples and state the main results.
- In lecture 2 we will give ideas of the proofs behind the equivalent characterizations of (strong) collapsed Anosov flows
- In lecture 3 we will explain some ideas behind the proofs that partially hyperbolic diffeomorphisms on hyperbolic 3-manifolds are collapsed Anosov flows, and explain how one can obtain accessibility in general
- In lecture 4, we will talk about how the examples build via the Bonatti–Gogolev–Hammerlindl–Potrie method lead to collapsed Anosov flows, and explain the construction of a new example of a collapsed Anosov flow isotopic to the identity. If time permits, we will discuss more generally results about self-orbit equivalences of Anosov flows.

References

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[FP2]   Fenley, S.; Potrie, R.; *Accessibility and ergodicity for collapsed Anosov flows*, arXiv:2103.14630