

# Lectures in Bedlewo: Methods for studying abelian actions and centralizers

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## Abstract

Given a dynamical system  $f : M \rightarrow M$ , where  $f \in \text{Diff}(M)$ , generic situation should be that the only smooth coordinate changes of  $M$  under which  $f$  does not change at all, are powers of  $f$ . This is Smale's conjecture, it is proved in  $C^1$  topology by Bonatti-Crovisier-Wilkinson. We will discuss here the non-generic situation, when the smooth centralizer  $Z(f) = \{g \in \text{Diff}(M) : g \circ f \circ g^{-1} = f\}$  of  $f$  is a larger group. The non-generic situation is in fact typical within the class of algebraic dynamical systems. Algebraic systems are defined via automorphisms and translations on homogeneous spaces  $G/\Gamma$ , where  $G$  is a Lie group and  $\Gamma$  a lattice. There has been a large body of work in the direction of showing that if  $Z(f)$  is sufficiently large for an Anosov  $f$ , then  $f$  is essentially algebraic (i.e.  $f$  is smoothly conjugate to an algebraic system). This started with the local result of Katok-Spatzier and was then continued in two general directions: obtaining global results while assuming enough Anosov dynamics in  $Z(f)$ , and in the direction of weakening the Anosov condition to partially hyperbolic and obtaining local or, more recently, semi-local results.

The mini-course will consist of 6 lectures where we will cover the following topics:

Lectures 1 and 2. Introduction into local and global rigidity for abelian actions and centralizer rigidity. Examples and basic invariant structures for abelian actions.

Lecture 3. Two methods: to linearize or not to linearize. Cohomology.

Lecture 4. Focusing on partially hyperbolic conservative dynamics, disintegration of volume along foliations, large centralizer vs pathological center foliation.

Lecture 5. Transitive centralizer and fibered partially hyperbolic systems.

Lecture 6. Focusing on accessible examples: centralizer rigidity and centralizer classification.