

# MINICOURS BEDLEWO: DIMENSION OF STATIONARY MEASURES

FRANÇOIS LEDRAPPIER AND PABLO LESSA

Abstract: The goal of these lectures is to present results and methods for studying the dimension of stationary measures of random walks. The setting will be the action of matrix groups over flag spaces. Exact dimension is proven for dynamical (Oseledets) measures. Other problems have only partial answers. A good part of the study is understanding the fine structure of the flag space.

- I. Generalities about dimension of measures, random walks on matrices. Statement of results (F).
- II. Strategy of the proof in dimension 3. Example of Hitchin representations (F).
- III. Admissible topologies on  $1, \dots, N$  and configuration spaces (P).
- IV. Change of coordinates and dynamics of configuration spaces (P).
- V. One-step formula (P).
- VI. Addition of entropy and dimension (F).

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