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On the algebraic structure of groups area-preserving homeomorphisms

In an influential article from the 1970s, Albert Fathi, having proven that the group of compactly supported volume-preserving homeomorphisms of the n -ball is simple for $n \geq 3$, asked if the same statement holds in dimension 2. In a joint work with Cristofaro-Gardiner and Humilière, we proved that the group of compactly supported area-preserving homeomorphisms of the 2-disc is not simple. This answers Fathi's question and settles what is known as "the simplicity conjecture" in the affirmative.

In fact, Fathi posed a more general question about all compact surfaces: is the group of "Hamiltonian homeomorphisms" (which I will define) simple? The main goal of these lectures will be to review recent joint work with Cristofaro-Gardiner, Humilière, Mak and Smith answering this more general question of Fathi. The solution relies on a collection of numerical invariants of area-preserving maps which we call **link spectral invariants**. In the first talk, I will show how these invariants lead to the solution of Fathi's question. The following talks will be dedicated to the construction of these invariants which relies on a version of Lagrangian Floer homology. We will be reviewing the construction of **Lagrangian Floer homology** and the associated spectral invariants.