Spectrum rigidity and joint integrability for Anosov systems on tori

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Abstract. In this minicourse, we address on the strong rigidity properties from joint integrability in the setting of Anosov diffeomorphisms on tori. More specifically, for an irreducible Anosov diffeomorphism with splitted stable bundle, the joint integrability of the strong stable and full unstable subbundles implies existence of fine dominated splitting along the weak stable subbundle as well as Lyapunov exponents rigidity. This builds an equivalence bridge between the geometric rigidity (joint integrability) and dynamical spectral rigidity (Lyapunov exponents rigidity) for Anosov diffeomorphisms on tori. Moreover, we will show that the topological conjugacy of non-invertible Anosov maps on 2-torus implies the smooth conjuacy along stable foliations.

The course will be split into four lectures:

1. We study local perturbation of generic irreducible linear automorphisms on $\mathbb{T}^d$ and prove the joint integrability of the strong stable and full unstable subbundles of these Anosov diffeomorphisms implies smooth conjugacy along weak stable bundles.

2. We show some global rigidity of irreducible Anosov diffeomorphisms on $\mathbb{T}^d$ satisfying center bunching condition. The joint integrability of the strong stable and full unstable subbundles of these Anosov diffeomorphisms implies dynamical coherence, full leaf conjugacy, and matching finest dominated splittings and spectral rigidity in weak stable bundles.

3. Let $A \in \mathrm{Sp}(4,\mathbb{Z})$ be an irreducible and non-conformal Anosov automorphism. For any symplectic diffeomorphism f which is C^1-close to A, we show that the extremal symplectic bundle of f is integrable if and only f is smoothly conjugate to A.

4. We show that if two non-invertible Anosov maps on 2-torus are topological conjugate, then they also admit spectral rigidity along stable bundles, i.e. they have the same Lyapunov exponents on corresponding periodic points. In particular, the conjugacy is smooth along stable foliations.

References

[1] J. An, S. Gan, R. Gu, Y. Shi, Rigidity of stable Lyapunov exponents and integrability for Anosov maps, arXiv:2205.13144.

[2] S. Gan, Y. Shi, Rigidity of center Lyapunov exponents and su-integrability, *Comment. Math. Helv.*, **95** (2020), no. 3, 569–592.

[3] A. Gogolev, Smooth conjugacy of Anosov diffeomorphisms on higher-dimensional tori, *J. Mod. Dyn.*, **2** (2008), no. 4, 645–700.

[4] A. Gogolev, M. Guysinsky, C^1-differentiable conjugacy of Anosov diffeomorphisms, on three dimensional torus, *Discrete Contin. Dyn. Syst.*, **22** (2008), no. 1–2, 183–200.

[5] A. Gogolev, B. Kalinin and V. Sadovskaya, Local rigidity for Anosov automorphisms, *Math. Res. Lett.*, **18** (2011), no. 5, 843–858.

[6]   A. Gogolev, Y. Shi,   Spectrum rigidity and jointly integrability for Anosov diffeomorphisms, *arXiv:2207.00704*.

[7] R. Gu, Y. Shi, Topological and smooth classification of Anosov maps on torus, *arXiv:2212.11457*.

[8]   A.Hammerlindl, Y.Shi, Accessibility of derived-from-Anosov systems, *Trans. Amer. Math. Soc.*, **374** (2021), no. 4, 2949–2966.

[9] F. Przytycki, Anosov endomorphisms, *Studia Math.*, **58** (1976), no. 3, 249–285.