
ON PAIRS OF COMPLEMENTARY TRANSMISSION CONDITIONS AND ON APPROXIMATION OF SKEW BROWNIAN MOTION BY SNAPPING-OUT BM

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We find the decomposition of Cartesian product of two copies of a space of continuous functions on the real line into the direct sum of two subspaces that are invariant under a cosine family of operators underlying Brownian motion. Both these subspaces are formed by pairs of extensions of continuous functions: in the first subspace the form of these extensions is shaped unequivocally by the transmission conditions

$$\begin{aligned}f'(0-) &= \alpha(f(0+) - f(0-)), \\f'(0+) &= \beta(f(0+) - f(0-)),\end{aligned}$$

describing snapping out Brownian motion, in the second, it is shaped by the transmission conditions of skew Brownian motion with certain degree of stickiness. In this sense, the above transmission conditions are complementary to each other.

As an application of the analysis leading to this result, we provide a deeper semigroup-theoretic insight into the theorem saying that as the coefficients α and β tend to infinity but their ratio remains constant, the snapping-out Brownian motions converge to a skew Brownian motion and we exhibit a corresponding complementary conditions.

References

- [1] A. Bobrowski and E. Ratajczyk, Pairs of complementary transmission conditions for Brownian motion, *Math. Ann.*, (2023), <https://doi.org/10.1007/s00208-023-02613-x>

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