Periodic solutions to symmetric Newtonian systems in neighborhoods of orbits of equilibria

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The aim of this talk is to discuss the existence of periodic solutions to Newtonian systems of the form

$$\ddot{u}(t) = -\nabla U(u(t)). \tag{1}$$

in neighborhoods of equilibria. Allowing the potential U to be symmetric, we consider equilibria which are not necessarily isolated. More precisely, if the potential U is Γ-invariant for a compact Lie group Γ, the equilibria form orbits of the action of this group. Consequently, if $\dim \Gamma \geqslant 1$, then it can happen that $\dim \Gamma(u_0) \geqslant 1$, i.e., the critical point u_0 is not isolated in $(\nabla U)^{-1}(0)$.

Assuming these orbits of equilibria to be isolated, we apply equivariant bifurcation techniques to obtain a generalization of the classical Lyapunov center theorem. Our tool is an equivariant version of the Conley index given in [1]. To compare the indices we compute cohomological dimensions of some orbit spaces. To this end we use the results given in [2].

The talk is based on the paper [3].

References

[1] M. Izydorek, *Equivariant Conley index in Hilbert spaces and applications to strongly indefinite problems*, *Nonlinear Anal.*, **51** (2002), 33–66.

[2] T. Kawasaki, *Cohomology of twisted projective spaces and lens complexes*, *Math. Ann.*, **206** (1973), 243–248.

[3] A. Gołębiewska, M. Kowalczyk, S. Rybicki, P. Stefaniak *Periodic solutions to symmetric Newtonian systems in neighborhoods of orbits of equilibria* Electron. Res. Arch. **30(5)** (2022), 1691–1707.

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