FINITE-DIMENSIONAL GLOBAL AND EXPONENTIAL ATTRACTORS FOR SOME FOURTH ORDER PROBLEMS IN $\mathbb{R}^N$

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A Cauchy problem for a dissipative fourth order parabolic equation in $\mathbb{R}^N$ is considered

$$u_t + \Delta^2 u = g(x) + m(x)u + f_0(x, u), \ t > 0, \ x \in \mathbb{R}^N,$$

$$u(0, x) = u_0(x) \in H^2(\mathbb{R}^N), \ x \in \mathbb{R}^N,$$

with a mildly integrable potential function $m\colon \mathbb{R}^N \to \mathbb{R}$ such that

$$\|m\|_{L^r_U(\mathbb{R}^N)} = \sup_{y \in \mathbb{R}^N} \|m\|_{L^r(B(y,1))} < \infty \tag{1}$$

for some $\max\left\{\frac{N}{4}, 1\right\} < r \leqslant \infty$. Using the quasi-stability method by Chueshov and Lasiecka an estimate from above of the fractal dimension of a global attractor is derived. It is also shown that the global attractor is contained in a finite-dimensional exponential attractor.

Applying the same method, we obtain similar results for the Cahn-Hilliard-Oono equation in $\mathbb{R}^N$:

$$u_t + \Delta(\Delta u + f(x, u)) + \delta u = 0, \quad t > 0, \ x \in \mathbb{R}^N, \ (\delta > 0)$$

$$u(0, x) = u_0(x) \in H^1(\mathbb{R}^N), \ x \in \mathbb{R}^N, \ (N \leqslant 3)$$

for example for

$$f(x, u) = m(x)u - u|u|^{\rho-1},$$

where $\|m\|_{L^r_U(\mathbb{R}^N)} < \infty$ with $2 \leqslant r \leqslant \infty$ and

$$\rho > 1 \text{ arbitrarily large if } N = 1, 2 \text{ and } 1 < \rho < \rho_c := \frac{N+2}{N-2} = 5 \text{ if } N = 3.$$

This is a joint work with Jan W. Cholewa based on the articles [1,2].

Suitably adapted considerations lead to corresponding results for a modified Swift-Hohenberg equation in $\mathbb{R}^N$ as was shown in cooperation with Maria Kania-Błaszczyk in papers [3,4].

[1] J.W. Cholewa, R. Czaja, On fractal dimension of global and exponential attractors for dissipative higher order parabolic problems in $\mathbb{R}^N$ with general potential, in *Contemporary Approaches and Methods in Fundamental Mathematics and Mechanics*, Understanding Complex Systems, Springer, 2021, 293–314.

[2] J.W. Cholewa, R. Czaja, Exponential attractor for the Cahn-Hilliard-Oono equation in $\mathbb{R}^N$, preprint.

[3] R. Czaja, M. Kania, Dissipative mechanism and global attractor for modified Swift-Hohenberg equation in $\mathbb{R}^N$, *Turkish J. Math.* 46 (2022), 2728–2750.

[4] R. Czaja, M. Kania, Exponential attractors for modified Swift-Hohenberg equation in $\mathbb{R}^N$, *Differential Integral Equations* 36 (2023), 347–366.

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