NON-HOLONOMIC CONTROLLABILITY FOR FLOWS

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This talk is partially based on the joint research with Joanna Janczewska, Marek Izydorek and Eugene Stepanov. We study controllability (chain recurrence) for C^∞ smooth flows on a C^∞ smooth connected compact manifolds or Euclidean spaces. Consider a system

$$\dot{x} = V(x), \qquad x \in M, \qquad \dim M = m \tag{1}$$

where M is a C^∞ – smooth manifold and assume that the following condition is satisfied.

Condition 1. There exist k vector fields $X_1, \ldots, X_k \in C^\infty(M \to TM)$ such that the following two conditions are satisfied.

1. System (1) is chain recurrent (for any $\varepsilon > 0$ and any point $p \in M$ there exists a $T > 0$ and an ε – solution $\varphi(t)$ of system (1) such that $\varphi(0) = \varphi(T) = p$).
2. The linear hull of the Lie algebra engendered by vector fields $X_0 = V, X_1, \ldots X_k$ coincides with the tangent space TM.

The last statement is also known as Hörmander condition.

Theorem 1. *Let Condition 1 be satisfied and M be a compact manifold. Then, for any $\varepsilon > 0$ and any points $p, q \in M$ there exits a $T > 0$ and continuous functions $\alpha_1, \ldots, \alpha_k : [0, \infty) \to [0, \varepsilon)$ such that the solution $x(t, p)$ of system*

$$\dot{x} = V(x) + \alpha_1(t) X_1(x) + \ldots + \alpha_k(t) X_k(x)$$

with initial conditions $x(0, p) = p$ is such that $x(T, p) = q$.

This theorem, considered for $V = 0$ is the the well-known Chow-Rashevskii Theorem. Our result improves the therem given in a book by V.Judjević for recurrent (but not chain-recurrent) flows. Moreover, we offer a *constructive* proof. In a non-compact case, the item 2 of Condition 1 must be replaced with a so-called uniform Hörmander condition (which will be given) and one must assume that all the considered vector fields are uniformly bounded together with all their derivatives. We discuss an opportunity to prove a C^0 'non-holonomic' Closing Lemma (and, also, a Connecting Lemma) in assumoptions of Theorem 1. The research was supported by Gdańsk University of Technology by the DEC 14/2021/IDUB/I.1 grant under the Nobelium -'Excellence Initiative - Research University' program.

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