Periodic and connecting orbits for Mackey–Glass type equations

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Abstract

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We consider delay differential equations of the form

$$y'(t) = -ay(t) + b\frac{y^k(t-1)}{1+y^n(t-1)}$$

with positive real parameters a, b, k, n. The case $k = 1$ is the Mackey–Glass equation which was introduced in 1977 by Michael Mackey and Leon Glass as a model for the feedback control of blood cells. Since then this particular equation attracted the attention of mathematicians interested in nonlinear dynamics and delay differential equations. Despite intensive research over the decades with a large number of analytical and numerical results, the dynamics is not understood yet.

For each $k \geqslant 1$ there are parameter values $b > a > 0$ such that, for sufficiently large n, stable periodic orbits exist. The periodic orbits can have complicated shapes.

In the case $k > 1$ an additional equilibrium point ξ^* arises from which there exist connecting orbits to zero and to the stable periodic orbit obtained in the first step. Heteroclinc connections are shown between periodic orbits. Moreover, in some cases, homoclinic orbits to ξ^* appear.

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