Compactness of nonlinear integral operators with discontinuous kernels

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We discuss compactness of nonlinear integral operators in the space of continuous functions. A result that is commonly used is for kernels $g(t,s)$ which are continuous in t for almost every s. However, there are many cases where the kernel has a jump across the line $s=t$, so that result does not apply. I give a result where the kernel can have jumps across a finite number of curves, graphs of continuous functions, $s=\varphi_k(t)$. These types occur in the study of ordinary differential equations with boundary conditions of local or nonlocal type. In the study of fractional differential equations, operators with singular kernels arise. I prove a compact embedding theorem for fractional integrals in order to give a new treatment for this case.

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