

Ergodic and mixing properties of horocycle flows and their time-changes

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Horocycle flows are the prime examples of homogeneous unipotent flows and a paradigm of “uniformly parabolic” dynamics. The simplest perturbations which preserve the parabolic behaviour are smooth time-changes. Despite their apparent simplicity, they display interesting and surprising phenomena, and several basic questions are still open.

In these six lectures, we will study the ergodic and mixing properties of horocycle flows and their smooth time-changes. A tentative program is structured as follows:

1. **The setting.** Definitions and basic constructions: homogeneous flows and the Haar measure. Action of $\mathrm{PSL}_2(\mathbb{R})$ on the hyperbolic plane. The Casimir operator.
2. **Ergodicity.** Precise asymptotics of horocycle ergodic averages on compact spaces.
3. **Mixing.** The “mixing via shearing” method and equidistribution of transverse curves. Ratner’s result on mixing rates.
4. **Time-changes.** Definitions and basic facts. Ergodicity and mixing.
5. **Rigidity.** Ratner’s Rigidity Theorem.
6. **Beyond $\mathrm{SL}_2(\mathbb{R})$.** Time-changes and other perturbations in different settings.