

# Introduction to KAM Theory

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The aim of this course is to present an introduction to some of the central ideas in the Kolmogorov-Arnold-Moser (KAM) theory on the persistence of quasi-periodic motions.

The main motivation for KAM theory comes from stability questions in celestial mechanics, a topic extensively studied by astronomers, physicists, and mathematicians in the last centuries. When approaching these problems in a perturbative fashion, it is sometimes possible to develop formal series expansions for their solutions. However, the convergence of these series is not easily established due to the presence of arbitrarily small divisors.

KAM theory proposes a way to overcome this formidable difficulty, and it is the purpose of these lectures to explain the fundamentals of this method.

With this in mind, we will first study in detail one of the simplest cases where these techniques can be applied, namely, the linearization problem of an analytic circle diffeomorphism close to a circle rotation. Then, we will discuss briefly the theory in its classical context, that of Hamiltonian dynamical systems.