Introduction to Smooth Ergodic Theory

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Abstract: One of the main goals of Ergodic Theory is to describe the statistical behavior of orbits of a given system. For dynamical systems exhibiting some form of hyperbolicity, certain dynamical foliations (stable/unstable foliations) prove to be very useful in the study of statistical properties of such systems. In this minicourse, I will recall basic notions of these foliations, and how these geometric objects interact with some invariant measures of particular interest, the so-called SRB measures, which describe the statistical behavior of a "large" set of points. In the special case of conservative systems, stable/unstable foliations can be used to prove properties such as ergodicity; I will recall a famous argument due to Hopf in this direction. More generally, for dissipative hyperbolic systems, "SRB measures are the invariant measures most compatible with volume when volume is not preserved" (quoting L.-S. Young); I will recall how they can be constructed, and their basic properties.