Random matrix products: stationary probability measures on the projective space.

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The goal of this course is to get familiar with the theory of random matrix products, which is the study of the long time behavior of a product $X_n \cdots X_1$ with the X_i's stationary random matrices of fixed dimension. As it is knwon since its initiation with Furstenberg, Kesten, Virtzer, Tutubalin in the 60-70's and its developments in the 70-90's with Guivarc'h, Raugi, Bougerol, Goldsheid, Margulis (etc.), the study of such products turn out to require a beautiful interplay between dynamical systems and ergodic theory, probability theory, algebraic groups and their subgroups. Its recent use in the context of homogeneous dynamics (starting with the seminal works of Benoist–Quint) opened new research lines.

In a first part of the course, we introduce and study the basic tools needed to study random matrix products: products (Lyapunov exponents, stationary measures, etc.) and study their properties. The ultimate goal is to give a full classification of the stationary probability measures on the projective space of the natural Markov chain induced by these products in the i.i.d case. This classification is the result of two joint works with C. Sert and builds a bridge between foundational results of Furstenberg–Kifer (82) and those of Guivarc'h–Raugi (07) and Benoist–Quint (16).