New equivariant dimensions

Ludwik Dąbrowski
SISSA, Trieste (I)

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Borsuk-Ulam Theorem

Here: spaces are Hausdorff & maps continuous,
$S^n \subset \mathbb{R}^{n+1}$ (unit sphere) and $B^n \subset \mathbb{R}^n$ (unit ball),
for any $n \in \mathbb{N}$, $\sigma : x \mapsto -x$ (antipodal involution of S^n).

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Theorem (B-U: 2 (of many) equivalent statements)

(i) $$\nexists\ f\colon S^n \to S^{n-1},\ f \circ \sigma = \sigma \circ f.$$

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$\looparrowright$ σ generates a free action of the group $\mathbb{Z}/2\mathbb{Z}$,
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Different proofs, lot of applications, various equivalent thms, plentiful corollaries (some "fun facts"). Also (ii) $\Rightarrow$

Theorem (Brouwer fixed-point theorem: equivalent statement)

$\nexists\ g : B^n \to S^{n-1}$ *such that* $g|_{\partial B^n} = id.$

i.e. S^{n-1} is not a retract of B^n, or contractible (since $B^n = \mathcal{C}S^{n-1}$).

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- Obviosity: $\mathbb{Z}/2\mathbb{Z}$ isn't contractible

Baum, D, Hajac formulated a generalized Borsuk-Ulam conjecture in terms of *topological join* of two spaces X and Y:

$$X * Y = X \times Y \times [0, 1]/\text{collapse } X \text{ at } 0 \text{ and } Y \text{ at } 1.$$

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Conjecture (BU)

Let X a compact space with free action of a compact group $G \neq 1$. Then, for the diagonal action of G on the topological join $X * G$, there is no G-equivariant map $X * G \to X$.

Since $\boxed{S^n * \mathbb{Z}/2\mathbb{Z} \simeq S^{n+1}}$ it generalizes BUT(i).

BU corollaries

$$
\begin{tikzcd}
\text{BU}: \nexists \text{ equiv } X * G \to X \arrow[r, Rightarrow] \arrow[d, Rightarrow, shift right] & \text{BU}\underline{\text{II}}: \nexists \text{ equiv } X * G \to G \text{ i.e. } X * G \text{ is nontriv. PFB} \arrow[d, Rightarrow] \\
\text{BU}': \nexists\, \gamma : CX \to X,\ \gamma|_X \text{ equiv} \arrow[u, Rightarrow, shift right] \arrow[d, Rightarrow] & G * G \text{ is nontriv. P.FB} \arrow[d, Rightarrow] \\
\text{Brouwer}': \nexists\, \gamma : CX \to X,\ \gamma|_X = id_X \text{ i.e. } X \text{ is noncontractible} \arrow[r, Rightarrow] & G \text{ is noncontractible}
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Ageev and Hilbert-Smith conjectures

BDH conjecture partially settles [CDT] ♠

Conjecture (Ageev)

There are no G-equivariant maps $\mu^m \to \mu^n$, $m > n$, between Menger compacta with a free action of a non-trivial zero-dimensional compact metric group G.

In turn it implies

Conjecture (weak Hilbert-Smith for p-adic actions)

For arbitrary prime p the group of p-adic integers $\mathbb{Z}_p$ cannot act freely on a connected topological manifold M with dim $M/\mathbb{Z}_p < \infty$

which reduces to $\mathbb{Z}_p$

Conjecture (weak Hilbert-Smith)

A locally compact group G acting freely and properly on a connected topological manifold M such that the orbit space M/G of G is finite dimensional, is a Lie group.

BU-type Thms

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A stronger (best so far) result takes cue from:
if X and Y are compact principal G-bundles
so is the join $X * Y$ for the diagonal G-action (free; loc. triv.?).

In particular for any compact group $G \neq 1$,
$G * G$ is a non-trivial principal G-bundle over $\mathcal{S}G$.
Indeed G-equivariant $G*G \to G \leftrightarrow$ *contraction* of G (impossible).
For example: $G = \mathbb{Z}/2\mathbb{Z}, \quad U(1), \quad SU(2)$
yield: $S^1 \to \mathbb{R}P^1,\ S^3 \to S^2,\ S^7 \to S^4$ (Hopf fibrations).

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♠ More generally:

Theorem (Bestvina, Edwards (unpubl); Chirvasitu, D, Tobolski)

For a compact group $G \neq 1$ there are no G-equivariant maps $G^{*(n+1)} \to G^{*(n)}$ for the diagonal G-actions.

Equivalently,

Definition

For a compact group G and a compact G-space X define

$$\mathrm{ind}_G(X) = \inf\{n : \exists\ G\text{-map } X \to G^{*(n+1)}\}.$$

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Assume the opposite: $\mathrm{ind}_G(X) = n$ and $\exists$ a G-map $X * G \to X$.
The following chain of G-maps contradicts our Thm.

$G^{*(n+1)} \hookrightarrow X * G * G^{(*n)} \to X * G^{*(n)} \dots X * G \to X \to G^{*(n)}$.

($\Leftarrow$) since $G^{*(n+1)} = G^{*(n)} * G$ & are loc. trivial PFB. □

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($\Leftarrow$) since $G^{*(n+1)} = G^{*(n)} * G$ & are loc. trivial PFB. $\square$

$\looparrowright$ *X cpct: $\mathrm{ind}_G(X) < \infty$ iff X is a loc. trivial principal G-bundle*

Theorem (equivalent)

[BDH] conjecture holds when X is a loc. trivial principal G-bundle.

Go quantum (noncommutative)

Quantum spaces, groups and PFB

Pass to the "quantum" (Gelfand-Naimark$^+$) generalization [BDH15]:

$X \rightsquigarrow$ unital C*-algebra A,

$G \rightsquigarrow$ compact quantum group (H, Δ), where the *coproduct*

$$\Delta : H \to H \otimes_{\min} H$$

is an injective $*$-homomorphism, s.t. the *cancellation laws* hold:

$$\{\Delta(g)(1\otimes h) | g, h \in H\}^{cls} = H \otimes_{\min} H = \{(g \otimes 1)\Delta(h) | g, h \in H\}^{cls}.$$

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Then we take a free action of (H, Δ) on A
($=$ a coaction of H on A), i.e. an injective unital $*$-homomorphism

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(1) $(\delta \otimes \mathrm{id}) \circ \delta = (\mathrm{id} \otimes \Delta) \circ \delta$ (coassociativity),

(2) $\{\delta(a)(1 \otimes h) | a \in A, h \in H\}^{cls} = A \otimes_{\min} H$ ([Podleś] counitality)

(3) $\{(a \otimes 1)\delta(b) | a, b \in A\}^{cls} = A \otimes_{\min} H$ ([Ellwood] freeness)

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Call such A a quantum principal H-bundle & write just H for (H, Δ).

Locally trivial quantum PFB

A quantum analogue of trivialization is:
a $*$-homomorphism $\Phi : H \to A$, which is equivariant, i.e.

$$\delta \circ \Phi = (\Phi \otimes \mathsf{id}) \circ \Delta,$$

in which case δ is called *trivial* and A called *trivial* H-bundle.

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But only recently [GHTW]: δ is called *local-trivial* if there are d H-equivariant $*$-homomorphisms

$$\rho_0, \dots, \rho_d : C_0((0,1]) \otimes H \to A,$$

such that $\sum_{j=0}^{d} \rho_j(\mathrm{t} \otimes 1) = 1$,

and $\dim_{\mathrm{LT}}(\delta) := \inf\{d\}$ is called *local-triviality dimension*.

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$\looparrowright$ δ is local-trivial iff $\dim_{\mathrm{LT}}(\delta) < \infty$.

$\looparrowright$ δ is trivial iff $\dim_{\mathrm{LT}}(\delta) = 0$.

$\looparrowright$ δ is not growing under equivariant C*-homorphisms.

$\looparrowright$ for $A = C(X), H = C(G)$, $X \curvearrowleft G : r$ and $\delta = r^T$,
$\dim_{\mathrm{LT}}(\delta) = \mathrm{ind}_G(X)$ and $\dim_{\mathrm{LT}}(\delta) < \infty$ iff X is a loc. triv. PFB.

Quantum BU conjecture

Consider the *equivariant join* C*algebra [D,Hadfield,Hajac15],[Baum,De Commer,Hajac13]:

$$\boxed{A\overset{\delta}{\circledast}H := \left\{x \in C([0,1]) \otimes A \underset{\min}{\otimes} H \,\middle|\, \mathrm{ev}_0(x) \in H,\ \mathrm{ev}_1(x) \in \delta(A)\right\}}$$

with the free action of the compact quantum group (H, Δ)

$$\mathrm{id}\otimes\Delta\colon\ C([0,1],A) \underset{\min}{\otimes} H \longrightarrow C([0,1],A) \underset{\min}{\otimes} H \underset{\min}{\otimes} H.$$

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Conjecture (QBU [Baum,D,Hajac 15])

Let A be a unital C-algebra with a free action δ of a non-trivial compact quantum group (H,Δ), and let $A\circledast^{\delta}H$ be the equivariant noncommutative join C*-algebra of A and H with the induced free action of (H,Δ). Then,*

$$\boxed{\nexists\ H\text{-equivariant } *\text{-homomorphism } A \longrightarrow A\circledast^{\delta}H}\,.$$

$$
\begin{tikzcd}[column sep=large, row sep=large]
\text{QBU: } \nexists \text{ equiv } A \to A \circledast H \arrow[r, Rightarrow, "\text{if } \exists \text{ character on } \Lambda"'] \arrow[d, Rightarrow, shift right] & \text{QBU II: } \nexists \text{ equiv } H \to A \circledast H \arrow[d, Rightarrow] \\
\text{QBU}': \nexists\, \gamma : A \to \mathcal{C}A,\ ev_1 \circ \gamma \text{ is equiv} \arrow[u, Rightarrow, shift right, "\text{if } \exists \varepsilon \text{ on } H"'] \arrow[d, Rightarrow] & H \circledast H \text{ is nontriv QPFB} \arrow[d, Rightarrow] \\
\text{QBrouwer}': \nexists\, \gamma : A \to \mathcal{C}A,\ ev_1 \circ \gamma = id|_A \arrow[r, Rightarrow] & H \text{ is noncontractible}
\end{tikzcd}
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i.e. A is not contractible

Towards QBUT

It is convenient to consider if A and H admit or not a character:

1. A *N & H N:* So far Conj. proved for $A = H = C^*_r F_n$, $n > 1$
2. A *Y & H N:* Free action impossible ?
3. A *N & H Y:* Conj. holds trivially: given a character χ on H and a hypothetical $\psi : A \to A \circledast^\delta H$, $\chi \circ ev_0 \circ \psi$ would be a character on A (even for $H = \mathbb{C}$)
   E.g. the irrational quantum 2-torus with standard action of $\mathbb{T}^2$, the matrix algebra $M_n(\mathbb{C})$, $n > 1$ with a free $\mathbb{Z}/n\mathbb{Z}$-action, the Cuntz algebras $\mathcal{O}_n$, $n > 1$ with the gauge action of $U(1)$.
4. A *Y & H Y:* is the 'mainstream'; it contains the "torsion case", and the case of finite quantum groups with $A = H$

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In the best so far result one of the assumptions employs $\dim_{\mathrm{LT}}$ and the proof uses

$$\looparrowright \qquad \dim_{\mathrm{LT}}(\mathrm{id} \otimes \Delta) = \dim_{\mathrm{LT}}(\delta) + 1, \text{ for } A \circledast^\delta H.$$

Quantum BU Thm.

Let A be a unital C*-algebra with a free action δ of a compact quantum group (H, Δ).

Theorem (D,Hajac,Neshveyev16;Gardella,Hajac,Tobolski,Wu18)

If H admits a torsion character or a non-trivial classical subgroup of characters whose induced action is locally trivial, then

$$\boxed{\nexists \text{ an } H\text{-equivariant } *\text{-homomorphism } A \to A \circledast^{\delta} H.} \tag{1}$$

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Applications: noncontractibility of:

the Toeplitz algebra $\mathcal{T}$ since admits a free action of $\mathbb{Z}/2\mathbb{Z} \subset U(1)$ (even though $K^{\bullet}(\mathcal{T}) = K^{\bullet}(B^2)$ and B^2 is contractible)

- q-deformation $C(G_q)$ of a compact connected semisimple Lie group G since it has a torus of characters
- a finite quantum group H satisfies QBUT for $(A, \delta) = (H, \Delta)$.

Other: [Chirvasitu,Passer18] $C_r^*\Gamma$ is noncontractible if Γ is a discrete group that satisfies Baum-Connes conjecture

- stronger (K-theoretic) nontriviality results under different assumptions, eg. for the reduced C*-algebra $C_r^*(F_n)$, $n > 1$.

Final remarks

QBU Conjecture remains still open.
[Chirvasitu,Passer18] observe that for its solution there may be no well-behaved invariant, e.g. for all free actions of finite perfect groups in contrast to finite abelian ones.
In particular the local triviality dimension and the spectral count may both change under θ-deformations, full vs. reduced C*-algebra, and finite quotients in the commutative situation.

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Anyway, the new local-triviality dimension [GHTW] plays here important role, and is worth studying further on its own.
Then the class of graph C*-algebras is a natural testing ground for its investigation.
Recall that Vaksman-Soibelman quantum spheres S_q^{2n+1} defined as quantum homogeneous spaces of the Woronowicz quantum $SU(n+1)$ groups, with the antipodal Z/2Z-action were shown to have $\dim_{\mathrm{LT}} < \infty$ by using their presentation as graph C*-algebras.

GAP-101086394

Objectives:

a) Compute, or determine the finiteness of, the local-triviality dimension of actions on graph and higher-rank graph C*-algebras.

b) Study actions on the Toeplitz algebra, the Cuntz algebra O_2, and the quantum lens spaces.

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First steps: extend and apply research methods developed in [GHTW] where several tangible computations of the local-triviality dimension of actions on graph C*-algebras has been carried out. Try to detect any pattern (?) relating it to the adjacency matrix.

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Warm up excercise :)

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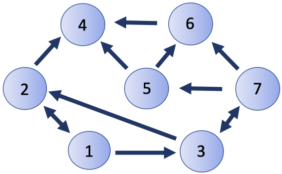

Impact

Achieving the above research goals would provide a deeper insight into the structure of graph C*-algebras. For instance, the finiteness of the local-triviality dimension of the $Z/2Z$-action on the Toeplitz algebra would mean that it contains a finite tuple of odd self-adjoint elements whose squares add up to one.
Moreover, these local-triviality computations directly aim at establishing quantum Borsuk–Ulam-type conjecture (our leitmotif) for group actions on graph C*-algebras.

The end

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&

keep $\dim_{\mathrm{LT}} > 0$