The rough Hawkes process

Donatien Hainaut*
UCLouvain, LIDAM
Maggie Chen†
Cardiff university, school of Mathematics
Enrico Scalas‡
Sussex university, Department of Mathematics

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Abstract

In this talk, we introduce a new class of self-excited jump processes, with a dampened rough (DR) kernel. The memory of this process is driven by the product of an exponential decreasing function and a kernel involved in the construction of the rough Brownian motion. This process, called rough Hawkes process, is nearly unstable since its intensity diverges to $+\infty$ for a very brief duration when a jump occurs. Firstly, we find the conditions that ensure the stability of the process and provide the closed form expression of the expected intensity. This process is not Markov, nevertheless we can reformulate the intensity as an infinite dimensional Markov process. Approaching these processes by discretization and next considering the limit leads to the Laplace's transform of the point process. This transform depends on the solution of an elegant fractional integro-differential equation. The fractional operator is defined by the DR kernel and is similar to the left-fractional Riemann-Liouville integral. We provide a simple method for computing the Laplace's transform. This is easily invertible by discrete Fourier's transform for retrieving the probability density of the process. We also modify the Ogata's algorithm to manage the instability of the process. We conclude by presenting the log-likelihood of the rough Hawkes process and fit it to hourly Bitcoin log-returns from the 9/2/18 to the 9/2/23.

Keywords: self-excited process, Hawkes process, point process.

*Corresponding author. Postal address: Voie du Roman Pays 20, 1348 Louvain-la-Neuve (Belgium). E-mail to: donatien.hainaut(at)uclouvain.be
†E-mail to: chenj60(at)cardiff.ac.uk
‡E-mail to: E.Scalas(at)sussex.ac.uk