General scaling limits for nearly unstable Hawkes processes

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Even though Hawkes processes have been studied for a long time, the study of functional limit theorems was only initiated ten years ago by [1]. The authors proved law of large numbers and central limit theorems for stable Hawkes processes ($\|\varphi\|_{L^1} < 1$) and derived few financial applications. Since then, Jaisson an Rosenbaum developed new method to investigate the nearly unstable case, $\|\varphi\|_{L^1} \to 1$. Meanwhile, pushed by neuroscience applications, [2,3] studied mean field limits for Hawkes processes

Here, we will take a step back and reconsider the assumptions of [4,5] to get more general results extending the class of nearly instable Hawkes processes. The limits are generalised Volterra processes of the form

$$Y(t) = K(t) + \int_0^t f(t - s)\sqrt{Y(s)}\, dB(s).$$

Then we extend this approach to consider mean field limits of nearly unstable Hawkes processes.

References

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