Multivariate self-exciting jump processes with applications to financial data

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The paper discusses multivariate self- and cross-exciting processes. We define a class of multivariate point processes via their corresponding stochastic intensity processes that are driven by stochastic jumps. Essentially, there is a jump in an intensity process whenever the corresponding point process records an event. An attribute of our modelling class is that not only a jump is recorded at each instance, but also its magnitude. This allows large jumps to influence the intensity to a larger degree than smaller jumps. We give conditions which guarantee that the process is stable, in the sense that it does not explode, and provide a detailed discussion on when particular subclasses, which include both linear and non-linear models, are stable. Finally, we fit our model to financial time series data from the S&P 500 and Nikkei 225 indices respectively. We conclude that a non-linear variant from our modelling class fits the data best. This supports the observation that in times of crises (high intensity) jumps tend to arrive in clusters, whereas there are typically longer times between jumps when the markets are calmer. We moreover observe more variability in jump sizes when the intensity is high, than when it is low.

Keywords: Multivariate self-exciting processes; Markov processes; Jump processes; Financial time series