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Title: Functional Limit Theorems for Hawkes Processes

Abstract: In this talk we introduce the full functional law of large numbers and functional central limit theorem for stationary or quasi-stationary Hawkes processes. Our results state that their asymptotic behaviors at a large time scale are totally determined by the average number and divergence of triggered events. Indeed, the classic functional central limit theorem with a Brownian motion modified by a power function holds for stationary Hawkes processes. In particular, the modification factor or the Brownian motion vanishes when triggered events are divergent or super-divergent respectively. For the quasi-stationary Hawkes processes with triggered events clustering around their mother events, the functional central limit theorem fails, but the scaling limit theorem holds with the limit process being the integral of CIR-model. Additionally, we also describe the convergence speed by establishing the upper bound between the distributions of rescaled Hawkes process and the corresponding limit process with respect to the Wasserstein distance. By contrary, because the divergence of triggered events results in the loss of self-excitation, the normalized quasi-stationary Hawkes process with divergent triggered events converges weakly to a Gaussian process with a nonlinear drift and long-range dependence.