The Weak AIM Conjecture

Michael Kinyon

A loop is said to be an AIM loop if it has an **A**belian **I**nner **M**apping group. Every nilpotent loop of nilpotency class at most 2 is an AIM loop, and the converse was an open problem up until the early 2000's. Counterexamples were constructed first by Csörgő and then by others, and now AIM loops of nilpotency class 3 are known as loops of Csörgő type.

For a loop Q, let $\mathrm{Nuc}(Q)$ denote the nucleus and let $Z(Q)$ denote the center. Examination of the structure of the known loops of Csörgő type led to a conjecture.

Strong AIM Conjecture: If Q is an AIM loop, then $Q/Z(Q)$ is a group and $Q/\mathrm{Nuc}(Q)$ is an abelian group.

The Strong AIM Conjecture implies the following.

Weak AIM Conjecture: If Q is an AIM loop, then Q is nilpotent of class at most 3.

I have given many talks about the Strong AIM Conjecture over the last 15 years. Although most work about it remains unpublished, it is known to hold in many classical varieties of loops. However, the conjecture in full generality remains open.

Bob Veroff, Petr Vojtěchovský and I have found a proof of the Weak AIM Conjecture. It is an interesting combination of automated deduction proofs and human reasoning involving isotopy. The proof also involves some lesser known varieties of loops, such as those with isotopically invariant flexibility and middle Bol loops.

In this talk, I will carefully outline the proof of the Weak AIM Conjecture. I will also discuss the current status of the Strong AIM Conjecture and why I think there is a pretty good chance it is false.