Quasi non-degenerate and quasi bijective set-theoretical solutions of the Yang-Baxter equation

Paola Stefanelli
University of Salento, Italy
paola.stefanelli@unisalento.it

In 1992, Drinfel'd [3] posed the question of determining all *set-theoretical solutions of the Yang-Baxter equation*, a fundamental equation of the statistical mechanics that arose from two independent works by Yang (1967) and Baxter (1972). Although several papers deal with this topic, a complete description of these solutions is still unknown.

In 2007, Rump [5] proved that involutive non-degenerate solutions correspond to *braces*, namely, algebraic structures that include Jacobson radical rings. The interest in braces and their generalizations has considerably grown in the last few years, and particular attention is devoted to the interplay between the properties of these structures and the behaviour of solutions associated to them.

In this talk, we present *dual weak braces*, structures recently introduced in [2] (see also [1]) that are triples $(S, +, \circ)$ with $(S, +)$ and $(S, \circ)$ Clifford semigroups satisfying the relations

$$a \circ (b + c) = a \circ b - a + a \circ c \qquad \text{and} \qquad a \circ a^{-} = a - a, \tag{$*$}$$

for all $a, b, c \in S$, where a^{-} and $-a$ are the inverses of a with respect to $+$ and $\circ$, respectively. If $(S, +)$ and $(S, \circ)$ are groups, then they have the same identity 0, and the second equality in $(*)$ is trivially satisfied; in this case, S is a *skew brace* [4] and, more specifically, if in addition $(S, +)$ is abelian, then S is a brace. We show that every dual weak brace gives rise to a solution near to being bijective and near to being non-degenerate. Finally, we focus on some properties of dual weak braces and their influence on solutions.

References

[1] F. Catino, M. Mazzotta, M.M. Miccoli, P. Stefanelli, *Set-theoretical solutions of the Yang-Baxter equation associated to weak braces*, Semigroup Forum **104** (2022), no. 2, 228 - 255.

[2] F. Catino, M. Mazzotta, P. Stefanelli, *Rota–Baxter operators on Clifford semigroups and the Yang–Baxter equation*, J. Algebra **622** (2023) 587 - 613.

[3] V. G. Drinfel'd, *On some unsolved problems in quantum group theory*, in: Quantum groups (Leningrad, 1990), vol. 1510 of Lecture Notes in Math., Springer, Berlin, (1992), pp. 1 - 8.

[4] L. Guarnieri, L. Vendramin, *Skew braces and the Yang-Baxter equation*, Math. Comp. **86** (307) (2017), 2519 - 2534.

[5] W. Rump, *Braces, radical rings, and the quantum Yang-Baxter equation*, J. Algebra **307** (1) (2007), 153 - 170.