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Metrics of positive curvature with conic singularities on compact surfaces

Let S be a compact Riemann surface, $A = (a_1, \dots, a_n)$ points in S and $\alpha_1, \dots, \alpha_n$ positive numbers. Consider Riemannian metrics of constant curvature κ on $S\backslash A$ with conic singularities with angles $2\pi\alpha_j$ at $a_j,\ 1 \le j \le n$. The main question is how to describe the set of all such metrics. These questions were completely settled in 19th century for $\kappa \le 0$, but are still wide open for $\kappa > 0$. The question is equivalent to description of the set of solutions of the differential equation

$$\Delta u + \kappa e^{2u} = 0$$

on $S\backslash A$ with prescribed singularities at the points of A. The subject is also related to geometry and analytic theory of linear ODE. A survey of recent results on this problem will be given.