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Counting periodic orbits of one-dimensional maps

For several decades, mathematicians have been fascinated with counting periodic orbits. It leads to somewhat unexpected connections to other areas of mathematics, like number theory, and thermodynamic formalism. This minicourse is an introduction to this subject, in the special case of one-dimensional maps.

We will start with a review of the work of Milnor and Thurston for interval maps. It concerns the (unweighted) counting of periodic orbits, the Artin-Mazur zeta function that encodes it, and its connection to the topological entropy. We will discuss open problems around the rationality of the Artin-Mazur zeta function, and the related ongoing work of Olivares-Vinales.

The remainder of the minicourse will be concerned with counting periodic orbits using weights, for interval and complex rational maps. In this setting, the counting of periodic orbits is encoded by a weighted version of the Artin-Mazur zeta function that was introduced by Ruelle. The study of the analytic properties of Ruelle's zeta function leads naturally to the pressure function and other objects from the thermodynamic formalism.

A highlight will be the prime orbit theorem established by Parry and Pollicott for uniformly hyperbolic maps, and its extension to non-uniformly hyperbolic maps in dimension 1 by Zhiqiang Li and expositor. The statement and proof of the prime orbit theorem are analogous to those of the prime number theorem in number theory. The extension to the non-uniformly hyperbolic case is based on the induced scheme developed by Feliks Przytycki and the expositor.