Stochastic Modeling and Control

May 8-13th, 2023

Abstracts

Affine processes driven by Lévy factors

Michał Barski
University of Warsaw
e-mail: mbarski@mimuw.edu.pl

The talk will be considered with an equation of the form

$$dR(t) = F(R(t))dt + \sum_{i=1}^{d} G_i(R(t-))dZ_i(t), \quad R(0) = x \geqslant 0, \quad t > 0, \quad (*)$$

where F, $G = (G_1, ..., G_d)$ are deterministic functions and $Z_1, ..., Z_d$ are independent Lévy processes. The solution R represents the short rate process and is supposed to generate bond prices $P(t,T)$ of the affine form, that is

$$P(t,T) = e^{-A(T-t)-B(T-t)R(t)}, \quad t \in [0,T],$$

where $A(\cdot), B(\cdot)$ are deterministic functions. In the case $d = 1$ all equations with this property were characterized in [2]. In the talk we discuss the multidimensional case. In particular, we point out the problem of a non-unique short rate representation which motivates introducing a classification of equations of the form $(*)$ in the spirit of Dai and Singleton [3]. The related canonical representations have the form

$$dR(t) = (aR(t) + b)dt + \sum_{k=1}^{g} c_k R(t-)^{1/\alpha_k} dZ_k^{\alpha_k}(t),$$

where $a, b, c_1, ..., c_g$ are some constants, $1 \leqslant g \leqslant d$ and $Z_k^{\alpha_k}$ is a stable Lévy process with index $\alpha_k \in (1,2]$. Numerical results on calibration of canonical models to market data, i.e. spot rates, Libor and swap rates will be presented.

The talk is based on a joint work with R. Łochowski [1].

[1] M. Barski, R. Łochowski, Classification and calibration of affine models driven by independent Lévy processes, *preprint, 2023*, https://arxiv.org/abs/2303.08477
[2] M. Barski, J. Zabczyk, On CIR equations with general factors, *SIAM J.Financial Mathematics*, 2020, 11,1,131–147,
[3] Q. Dai, K. Singleton, Specification Analysis of Affine Term Structure Models, *The Journal of Finance*, 2000, 5, 1943-1978

Prediction problems and second order equations

Erhan Bayraktar
University of Michigan
e-mail: erhan@umich.edu

We study the long-time regime of the prediction problem in both full information and adversarial bandit feedback setting. We show that with full information, the problem leads to second order parabolic partial differential equations in the Euclidian space. We exhibit solvable cases for this equation. In the adversarial bandit feedback setting, we show that the problem leads to a second order equation in the Wasserstein space. Based on joint works with Ibrahim Ekren

and Xin Zhang [1, 2, 3, 4].

[1] E. Bayraktar, I. Ekren, Y. Zhang, On the asymptotic optimality of the comb strategy for prediction with expert advice, *Annals of Applied Probability*, 2020, 30 (6), 2517–2546
[2] E. Bayraktar, I. Ekren, X. Zhang, Finite-Time 4-Expert Prediction Problem, *Communcations in Partial Differential Equations*, 2020, 45 (7), 714–757
[3] E. Bayraktar, I. Ekren, X. Zhang, Prediction against limited adversary, *Journal of Machine Learning Research*, 2021, 22(72), 1–33
[4] E. Bayraktar, I. Ekren, X. Zhang, A smooth variational principle on Wasserstein space, *Proceedings of the AMS*, 2022, to appear

Mean Field Markov Decision Processes

Nicole Bäuerle
Karlsruhe Institute of Technology
e-mail: baeuerle@kit.edu

We consider mean-field control problems in discrete time with discounted reward, infinite time horizon and compact state and action space. The existence of optimal policies is shown and the limiting mean-field problem is derived when the number of individuals tends to infinity. Moreover, we consider the average reward problem and show that the optimal policy in this mean-field limit is ε-optimal for the discounted problem if the number of individuals is large and the discount factor close to one. This result is very helpful, because it turns out that in the special case when the reward does only depend on the distribution of the individuals, we obtain a very interesting subclass of problems where an average reward optimal policy can be obtained by first computing an optimal measure from a static optimization problem and then achieving it with Markov Chain Monte Carlo methods. We give two applications: Avoiding congestion on a graph and optimal positioning on a market place which we solve explicitly. This part of the talk is based on [1]. If time allows we will also discuss a computational approach for the discounted cost problem based on a small sample of the large population developed in [2].

[1] N. Bäuerle, Mean Field Markov Decision Processes, *To appear in: Applied Mathematics and Optimization*, 2023, ArXiv:2106.08755
[2] E. Bayraktar, N. Bäuerle, A.D. Kara, Finite approximations and Q learning for mean field type multi agent control, *preprint*, 2023, ArXiv:2211.09633

Dynamic Robo-advising Using Model Predictive Control

Tomasz R. Bielecki
Illinois Institute of Technology
e-mail: tbielecki@iit.edu

Robo Advisors (RAs) are fully- or semi-automated portfolio management systems for use by individual investors. These systems are becoming an increasingly wide-spread and important component of wealth and asset management industry. The key element of any robo-advising system is the portfolio selection (asset allocation or management) methodology. We propose a dynamic (multi-period) framework for designing the asset allocation engine for a RA. Our proposed

approach is rooted in Model Predictive Control (MPC). Developed for engineering applications MPC has proven to be a powerful tool for dealing with all sorts of dynamic decision problems under uncertainty (stochastic control problems). Typically, MPC does not return optimal solutions to a stochastic control problem, but a sub-optimal solutions. Nevertheless, it has been proven time and time again in various applications of MPC that this methodology provides highly satisfactory performance. The essence of MPC is to convert a stochastic control problem to a deterministic control problem by replacing future values of relevant random quantities by their forecasts, and to approximate an optimal control for a stochastic control problem with a sequence of optimal controls for a sequence of appropriately constructed deterministic control problems. Thus, one of the key ingredients of any MPC algorithm is the forecasting module. In our approach we propose a combination of Hidden Markov Model and Black-Litterman methodologies to generate forecasts of mean asset returns and covariances between assets returns in the portfolio.

This is joint work with Tao Chen (University of Michigan, USA), Igor Cialenco (Illinois Institute of Technology, USA) and Areski Cousin (University of Strasbourg, France).

Correlated equilibria and mean field games

Luciano Campi
University of Milan
e-mail: luciano.campi@unimi.it

In the context of mean field games (MFGs), we introduce a generalization of mean field game solution, called correlated solution, which can be seen as the mean field game analogue of a correlated equilibrium. The latter is a generalization of Nash equilibrium for stochastic games. Our notion of solution can be justified in two ways for MFGs in discrete time and finite state space: correlated solutions arise as limits of exchangeable correlated equilibria in restricted (Markov open-loop) strategies for the underlying N-player games, and approximate N-player correlated equilibria can be constructed starting from a correlated solution to the mean field game. Moreover, those results can be extended to progressive deviations, possibly depending on the whole history of the state and the flow of measures. In this talk we will focus especially on a further extension to continuous time MFGs through the notion of coarse correlated equilibrium. This talk is based on joint works with O. Bonesini, F. Cannerozzi and M. Fischer.

Optimal reinsurance via BSDEs in a partially observable model with jump clusters

Claudia Ceci
University of Rome SAPIENZA
e-mail: claudia.ceci@uniroma1.it

We investigate the optimal reinsurance problem when the loss process exhibits jump clustering features and the insurance company has restricted information about the loss process. We maximize expected exponential utility of terminal wealth and show that an optimal solution exists. By exploiting both the

Kushner-Stratonovich and Zakai approaches, we provide the equation governing the dynamics of the (infinite-dimensional) filter and characterize the solution of the stochastic optimization problem in terms of a BSDE, for which we prove existence and uniqueness of solution. After discussing the optimal strategy for a general reinsurance premium, we provide more explicit results in some relevant cases. The talk is based on [1].

[1] M. Brachetta, G. Callegaro, C. Ceci, C. Sgarra, Optimal reinsurance via BSDEs in a partially observable model with jump clusters, *preprint*, ArXiv:2207.05489

How random horizon affect the mathematical structures in optimal stopping problem and RBSDEs

Tahir Choulli
University of Alberta
e-mail: tchoulli@ualberta.ca

In this talk, we consider a pair $(\mathbb{F}, \tau)$, where $\mathbb{F}$ is the flow of information and τ is a random time which might not be an $\mathbb{F}$-stopping time. To this pair, we associate the new filtration $\mathbb{G}$ using the progressive enlargement of $\mathbb{F}$ with τ. For this setting governed with $\mathbb{G}$, we analyze the optimal stopping problem in many aspects. Besides characterizing the existence of the solution to this problem in terms of $\mathbb{F}$, we derive the mathematical structures of the value process of this control problem, and we single out the optimal stopping problem under $\mathbb{F}$ associated to it. These quantify somehow the impact of τ on the optimal stopping problem. As an application, we assume $\mathbb{F}$ is generated by a Brownian motion W, and we address the following linear reflected-backward-stochastic differential equations (RBSDE hereafter for short),

$$\begin{cases} dY_t = f(t)d(t\wedge\tau) + Z_t dW_{t\wedge\tau} + dM_t - dK_t, \quad Y_\tau = \xi, \\ Y \geqslant S \quad \text{on} \quad [0,\tau[, \quad \displaystyle\int_0^\tau (Y_{s-} - S_{s-})dK_s = 0 \quad P\text{-a.s.}. \end{cases}$$

For this RBSDE, we focus on answering the following problems: a) What are the sufficient minimal conditions on the data (f, ξ, S, τ) that guarantee the existence of the solution of the $\mathbb{G}$-RBSDE in L^p $(p > 1)$? b) How can we estimate the solution in norm using the triplet-data (f, ξ, S)? c) Is there an RBSDE under $\mathbb{F}$ that is intimately related to the current one and how their solutions are related to each other? The talk is based on [1].

[1] Alsheyab Safa and Tahir Choulli, Optimal Stopping Problem under random horizon" Mathematical structures and Linear RBSDEs, *2023*, Preprint available on Arxiv.
[2] Alsheyab Safa and Tahir Choulli, Reflected backward stochastic differential equations under stopping with an arbitrary random time, *2022*, Preprint available on Arxiv.

Learning to reflect – Data-driven solutions to singular control problems

Sören Christensen
Kiel University
e-mail: christensen@math.uni-kiel.de

While theoretical solutions to many stochastic control problems are well understood, their practicality often suffers from the assumption of known dynamics of the underlying stochastic process, which raises the statistical challenge of developing purely data-driven controls in a nonparametric framework.

In this talk, we discuss how to bring together stochastic control and statistics, which we explore for ergodic singular control problems and underlying one- and multi-dimensional diffusions. The exploration vs. exploitation dilemma plays an essential role in the considerations. We find exact rates of convergence of ponomial order for the regret and compare the results with those of deep Q-learning algorithms.

The talk is based on [1] and [2].

[1] S. Christensen, Claudia Strauch, Nonparametric learning for impulse control problems, *Annals of Applied Probability*, 2023, 33 (2), 1369 - 1387
[2] S. Christensen, Claudia Strauch, Lukas Trottner, Learning to reflect: A unifying approach for data-driven stochastic control strategies, *preprint*, arXiv:2104.11496

Risk Filtering and Risk-Averse Control of Systems with Model Uncertainty

Igor Cialenco
Illinois Institute of Technology
e-mail: cialenco@iit.edu

We consider a Markov decision process subject to model uncertainty in a Bayesian framework, where we assume that the state process is observed but its law is unknown to the observer. In addition, while the state process and the controls are observed at time t, the actual cost that may depend on the unknown parameter is not known at time t. The controller optimizes these running costs by using a family of special risk measures, that we call risk filters and that are appropriately defined to take into account the model uncertainty of the controlled system. These key features lead to non-standard and non-trivial risk-averse control problems, for which we derive the Bellman principle of optimality. We illustrate the general theory on several practically important examples.

This is joint work with Tomasz R. Bielecki (Illinois Tech, USA) and Andrzej Ruszczynski (Rutgers University, USA).

Signature methods in stochastic portfolio theory

Christa Cuchiero
University of Vienna
e-mail: christa.cuchiero@univie.ac.at

In the context of stochastic portfolio theory we introduce a novel class of portfolios which we call linear path-functional portfolios. These are portfolios which are determined by certain transformations of linear functions of a collection of feature maps that are non-anticipative path functionals of an underlying semimartingale. As main example for such feature maps we consider (random) signature of the (ranked) market weights. We prove that these portfolios are universal in the sense that every continuous, possibly path-dependent, portfolio function of the market weights can be uniformly approximated by signature portfolios. We also show that signature portfolios can approximate the log-optimal portfolio in generic classes of non-Markovian models arbitrarily well and illustrate numerically that the trained signature portfolios are remarkably close to the theoretical log-optimal portfolios.

Besides these universality features, the main numerical advantage lies in the fact that several optimization tasks like maximizing expected logarithmic utility or mean-variance optimization within the class of linear path-functional portfolios reduces to a convex quadratic optimization problem, thus making it computationally highly tractable. We apply our method to real market data demonstrating out-performance on out-of-sample data also in the presence of transaction costs.

The talk is based on joint work with Janka Moeller.

A quickest detection problem with false negatives

Tiziano De Angelis
University of Torino and Collegio Carlo Alberto
e-mail: tiziano.deangelis@unito.it

We formulate and solve a quickest detection problem with false negatives. A standard Brownian motion acquires a drift at an independent exponential random time which is not directly observable. Based on the observation in continuous time of the sample path of the process, an optimiser must detect the drift as quickly as possible after it has appeared. The optimiser can inspect the system multiple times upon payment of a fixed cost per inspection. If a test is performed on the system *before* the drift has appeared then, naturally, the test will return a negative outcome. However, if a test is performed *after* the drift has appeared, then the test may fail to detect it and return a false negative with probability $\epsilon \in (0,1)$. The optimisation ends when the drift is eventually detected. The problem is formulated mathematically as an optimal multiple stopping problem and it is shown to be equivalent to a recursive optimal stopping problem. Exploiting such connection and free boundary methods we find explicit formulae for the expected cost and the optimal strategy. We also show that when $\epsilon = 0$ our expected cost coincides with the one in Shiryaev's classical optimal detection problem.

The talk is based on joint work [1] with Quan Zhou and Jhanvi Garg (Texas A&M University).

[1] T. De Angelis, J. Garg, Q. Zhou, A quickest detection problem with false negatives, *preprint*, ArXiv:2210.01844

Solving partial differential equations with neural networks with application to pricing of insurance claims

Łukasz Delong
SGH Warsaw School of Economics
e-mail: lukasz.delong@sgh.waw.pl

We discuss how systems of partial differential equations can be solved numerically with neural networks. As an application we consider pricing of equity-linked life insurance contracts with death and survival benefits in a model with multiple stochastic risk factors: interest rate, equity, volatility, unsystematic and systematic mortality. We price equity-linked contracts by assuming that the insurer hedges the risks to reduce the local variance of the net asset value process and requires a compensation for the non-hedgeable part of the liability in the form of an instantaneous standard deviation risk margin. The price can then be expressed as a solution to a system of non-linear partial differential equations. We reformulate the problem as a backward stochastic differential equation with jumps and solve it numerically by the use of efficient neural networks. The solution method is next generalized so that it can be applied to a more general class of systems of non-linear partial differential equations in pricing problems in insurance and finance.

Equlibrium Investment with Random Risk Aversion

Sascha Desmettre
Johannes Kepler University Linz
e-mail: sascha.desmettre@jku.at

We solve the problem of an investor who maximizes utility but faces random preferences. We propose a problem formulation based on expected certainty equivalents. We tackle the time-consistency issues arising from that formulation by applying the equilibrium theory approach. To this end, we provide the proper definitions and prove a rigorous verification theorem. We complete the calculations for the cases of power and exponential utility. For power utility, we illustrate in a numerical example, that the equilibrium stock proportion is independent of wealth, but decreasing in time, which we also supplement by a theoretical discussion. For exponential utility, the usual constant absolute risk aversion is replaced by its expectation. The talk is based on [1].

[1] S. Desmettre, M. Steffensen, Equilibrium Investment with Random Risk Aversion, *Mathematical Finance*, 2023, 1-30, https://doi.org/10.1111/mafi.12394

Constrained nonzero-sum Markov games

François Dufour
Bordeaux INP and INRIA and Institut Mathématiques de Bordeaux
e-mail: francois.dufour@math.u-bordeaux.fr

In this talk, we consider a nonzero-sum Markov game on an abstract measurable state space with compact metric action spaces. The goal of each player is to maximize his respective payoff function under the condition that some constraints on a payoff are satisfied. We are interested in the existence of a Nash or noncooperative equilibrium. Under suitable conditions, which include absolute continuity of the transitions with respect to some reference probability measure, and continuity in action of the payoff functions and the density function of the transitions of the system, we establish the existence of a constrained stationary Markov Nash equilibrium, that is, the existence of stationary Markov strategies for each of the players yielding an optimal profile within the class of all history-dependent profiles. This a joint work with T. Prieto-Rumeau.

On Stochastic Control and (Mean Field) Games in Infinite Dimension

Fausto Gozzi
Luiss University, Roma, Italy
e-mail: fgozzi@luiss.it

We study a family of stochastic control/games arising in typical applications (e.g. boundary control and control of delay equations with delay in the control) with the ultimate aim of finding solutions of the associated HJB equations, regular enough to find optimal feedback controls (or closed-loop Nash equilibria). The main difficulty lies in the fact that the image of the control operator goes out of the state space (what we call unboundedness). This prevents the use of known methods to attack the problem. We expose the main ideas to prove our results on the HJB equation and the feedback controls. This is joint work with Federica Masiero. We also discuss applications of our results to concrete problems and briefly present going on research which applies our results to Mean Field Games and Mean Field Control. Joint projects with Salvatore Federico, Daria Ghilli, Andrzej Swiech.

[1] F. Gozzi, F. Masiero, Stochastic Control Problems with Unbounded Control Operators: solutions through generalized derivatives, *Siam J. on Control and Optimization*, 2023, to appear

Stochastic Impulse Control with Delay and Random Coefficients

Said Hamadene
Le Mans University
e-mail: hamadene@univ-lemans.fr

In this talk we discuss an infinite horizon impulse control problem with execution delay when the dynamics of the system is described by a general stochastic process adapted to the Brownian filtration. The problem is solved by means of probabilistic tools relying on the notion of Snell envelope and infinite horizon reflected backward stochastic differential equations. This allows us to establish the existence of an optimal strategy over all admissible strategies. We consider also the case of exponential utilities. The talk is based on [1].

[1] Djehiche, B., Hamadene, S., Hdhiri, I., & Zaatra, H., Infinite horizon stochastic impulse control with delay and random coefficients, *Mathematics of Operations Research*, 2022, 47(1), 665-689.

On bivariate distributions of local time of Itô-McKean diffusions

Jacek Jakubowski
University of Warsaw
e-mail: j.jakubowski@mimuw.edu.pl

For an Itô-McKean diffusion X let L denote its local time at 0. We present the new explicit description of the distribution of L_t in terms of convolution exponent and use the excursion theory to describe the bivariate distributions of the pair (X_t, L_t). We provide a simple connection formula for the distribution of excursions of a bivariate Itô-McKean diffusion from a hyperplane. Examples involving the distribution of local time are presented including a formula for the distribution of (X_t, L_∞) for a transient diffusion. The talk is based on [1].

[1] J. Jakubowski, M. Wiśniewolski, On bivariate distributions of local time of Itô-McKean diffusions, *Bernoulli Journal*, to appear 2023

Time-consistency in the mean-variance problem: A new perspective

Anna Jaśkiewicz
Politechnika Wrocławska
e-mail: anna.jaskiewicz@pwr.edu.pl

The talk is based on joint paper with Nicole Bäuerle [1]. We investigate discrete-time mean-variance portfolio selection problems viewed as a Markov decision process. We transform the problems into a new model with deterministic transition function for which the Bellman optimality equation holds. In this way, we can solve the problem recursively and obtain a time-consistent solution, that is,

an optimal solution that meets the Bellman optimality principle. We apply our technique for solving explicitly a more general framework and are able to solve some examples from [2].

[1] N. Bäuerle, A. Jaśkiewicz, Time-consistency in the mean-variance problem: A new perspective, *preprint*, ArXiv:2301.11218
[2] T. Björk, M. Khapko, A. Murgoci, Time-inconsistent control theory with finance applications, *Springer Nature Switzerland*, 2021

Impulse control with generalised discounting

Damian Jelito
Jagiellonian University and Institute of Mathematics PAS
e-mail: damian.jelito@uj.edu.pl

In recent years, control problems with generalised discounting have attracted growing attention in the literature. This can be linked to the observation that classical exponential discount function does not properly reflect decision-makers' behaviour. In this talk we discuss the effects of applying generalised discounting on a long-run impulse control problem for a Feller-Markov process. We show that the optimal value of the discounted problem is the same as the optimal value of its undiscounted version. Next, we prove that an optimal strategy for the undiscounted discrete time functional is also optimal for the discrete time discounted criterion and nearly optimal for the continuous time discounted one. This shows that the discounted problem is time-consistent and instead of a complex time-dependent Bellman equation one may consider its simpler time-independent version. The talk is based on a joint work with Łukasz Stettner.

Should I invest in the market portfolio? A parametric approach

Jan Kallsen
Kiel University
e-mail: kallsen@math.uni-kiel.de

This study suggests a parsimonious stationary diffusion model for the dynamics of stock prices relative to the entire market. Its aim is to contribute to the question how to choose the relative weights in a diversified portfolio and, in particular, whether the market portfolio behaves close to optimally in terms of the long-term growth rate. Specifically, we introduce the *elasticity bias* as a measure of the market portfolio's suboptimality. We heavily rely on the observed long-term stability of the capital distribution curve, which also served as a starting point for the *Stochastic Portfolio Theory* in the sense of Fernholz.

[1] R. Fernholz, Stochastic Portfolio Theory, *Springer*, 2002

Nonlinear reflected BSDEs and their applications to Dynkin games: beyond the semimartingale framework

Tomasz Klimsiak
Nicolaus Copernicus University and Institute of Mathematics PAS
e-mail: tomas@mat.umk.pl

Suppose we are given a probability space $(\Omega, \mathcal{F}, P)$ equipped with a right-continuous filtration $\mathbb{F} = \{\mathcal{F}_t, t \geqslant 0\}$ that satisfies usual conditions, and (possibly infinite) $\mathbb{F}$-stopping time T. Furthermore, consider an $\mathcal{F}_T$-measurable random variable ξ, a $Prog(\mathbb{F}) \otimes \mathcal{B}(\mathbb{R})$-measurable function $\Omega \times \mathbb{R}_+ \times \mathbb{R} \ni (\omega, t, y) \mapsto f(\omega, t, y) \in \mathbb{R}$, and $\mathbb{F}$-adapted càdlàg processes L, U of class (D) that satisfy $L \leqslant U$ and $\overline{\lim}_{a\to\infty} L_{T\wedge a} \leqslant \xi \leqslant \underline{\lim}_{a\to\infty} U_{T\wedge a}$. The talk is devoted to a link between Reflected BSDEs and nonlinear Dynkin games:

$$\operatorname{esssup}_{\sigma\geqslant\alpha} \operatorname{essinf}_{\tau\geqslant\alpha} \mathbb{E}^f_{\alpha,\tau\wedge\sigma}\Big(L_\sigma \mathbf{1}_{\{\sigma<\tau\}} + U_\tau \mathbf{1}_{\{\tau\leqslant\sigma<T\}} + \xi \mathbf{1}_{\{\sigma=\tau=T\}}\Big) \quad (1)$$

where $\mathbb{E}^f$ is the nonlinear expectation generated by f. There is an extensive literature on this link but under additional assumption that L, U satisfy Mokobodzki's condition. The case when the said condition is not imposed on the barriers is poorly studied.

It is known that in some instances the family of random variables given by (1) may be aggregated to a càdlàg process even if Mokobodzki's condition does not hold, and so, there arises a natural question whether such a process is related to some (unique) solution to RBSDE. Our goal is to extend the notion of RBSDEs and provide one-to-one correspondence between solutions of RBSDEs and value processes in Dynkin games (1).

[1] T. Klimsiak, Non-semimartingale solutions of reflected BSDEs and applications to Dynkin games, *Stochastic Process. Appl.*, 2021, 134, 208–239

Risk-sensitive Dynkin games with heterogeneous Poisson random intervention times

Gechun Liang
Warwick University
e-mail: g.liang@warwick.ac.uk

We solve constrained Dynkin games with risk-sensitive criteria, where two players are allowed to stop at two independent Poisson random intervention times, via the theory of backward stochastic differential equations. This generalizes the previous work of [Liang and Sun, Dynkin games with Poisson random intervention times, SIAM Journal on Control and Optimization, 2019] from the risk-neutral criteria and common signal times for both players to the risk-sensitive criteria and two heterogenous signal times. Furthermore, we establishe a connection of such constrained risk-sensitive Dynkin games with a class of stochastic differential games via Krylov's randomized stopping technique.

Stochastic equations driven by fractional processes in Banach spaces

Bohdan Maslowski
Charles University, Prague
e-mail: Bohdan.Maslowski@mff.cuni.cz

This talk is based on results obtained jointly with Petr Čoupek, Martin Ondreját, and Pavel Kříž, cf. [1], [2]. Initially, the problem of integration of temporal functions taking values in Banach spaces with respect to (possibly non-Gaussian) fractional processes is discussed. The family of fractional processes that is considered includes, for example, fractional Brownian motions for any Hurst parameter, the Rosenblatt process or, more generally, the Hermite processes. The class of Banach spaces satisfying our conditions includes, for example, the Lebesgue spaces, Sobolev spaces, or, more generally, the Besov and Lizorkin-Triebel spaces. Domains of the Wiener integrals are characterized for both scalar and cylindrical fractional processes. In general, the integrands take values in the space of gamma-radonifying operators from a certain homogeneous Sobolev-Slobodeckii space into the considered Banach space. An equivalent characterization in terms of a pointwise kernel of the integrand is also given. These results are applied to linear stochastic equations. In particular, solutions of the heat equation with distributed noise of low time regularity and of the heat equation with boundary noise are studied. Existence of weak solutions for semilinear equations is also considered. A negative result for the Rosenblatt process is shown.

[1] P. Čoupek, B. Maslowski and M. Ondreját, Stochastic integration with respect to fractional processes in Banach spaces, *J. Funct. Anal.*, 2022, 288(8), 109393, 62 pp.
[2] P. Čoupek, P. Kříž and B. Maslowski, Parameter estimation and singularity of laws on the paths space for SDEs driven by Rosenblatt process, *preprint*,

Standard and fractional Bessel and Cox-Ingersoll-Ross processes

Yuliya Mishura
Taras Shevchenko National University of Kyiv,
Mälardalen University, Västerås, Sweden
e-mail: yuliyamishura@knu.ua, yuliia.mishura@mdu.se

We condider Cox-Ingersoll-Ross and Bessel processes driven by the standard Wiener process, especially in the case when the number of degrees of freedom is small and the behaviour of the process becomes more irregular. Stochastic representation of the reflection terms is provided. Fractional counterpart is also studied. In this case the number of degrees of freedom is not so important and even is not defined, and the behavior of the process crucially depends on the value of the Hurst index of fBm. The talk is based on the papers [1-3].

[1] Y. Mishura, A. Yurchenko-Tytarenko, Fractional Cox-Ingersoll-Ross process with non-zero "mean", *Modern Stochastics: Theory and Applications*, 2018, 5(1), 99–111
[2] Y. Mishura, A. Yurchenko-Tytarenko, Fractional Cox-Ingersoll-Ross process with small Hurst indices, *Modern Stochastics: Theory and Applications*, 2019, 6(1), 13–39

[3] Y. Mishura, A. Yurchenko-Tytarenko, Standard and fractional reflected Ornstein-Uhlenbeck processes as the limits of square roots of Cox-Ingersoll-Ross processes, *Stochastics*, 2022

Multivariate Hawkes processes with graphs

Mariusz Niewęgłowski
Warsaw University of Technology
e-mail: mariusz.nieweglowski@pw.edu.pl

A very interesting and important class of stochastic processes was introduced by Alan Hawkes in [1]. These processes, called now Hawkes processes, are meant to model self-exciting and mutually-exciting random phenomena that evolve in time. The self-exciting phenomena are modeled as univariate Hawkes processes, and the mutually-exciting phenomena are modeled as multivariate Hawkes processes. In this talk we provide some results on markovianity of the Generalized Multivariate Hawkes Processes (GMHP) introduced in our earlier papers. GMHP are multivariate marked point processes that add an important feature to the family of the (classical) multivariate Hawkes processes: they allow for explicit modelling of simultaneous occurrence of excitation events coming from different sources, i.e. caused by different coordinates of the multivariate process. We propose that this structure of mutual excitations is specified in terms of the excitation graph. We provide results which show that under some conditions on its kernels the intensities of GMHP's are functions of a Markov processes. Moreover we show that it is possible to compute their Laplace transform by means of system of ODE's. The talk is based on [4].

[1] A.G. Hawkes, Spectra of Some Self-Exciting and Mutually Exciting Point Processes., *Biometrika*, 58(1):83–90, 1971.
[2] T.R. Bielecki, J. Jakubowski, M. Niewegłowski, Construction and Simulation of Generalized Multivariate Hawkes Processes, *Methodology and Computing in Applied Probability*, (2022) 24:2865–2896
[3] T.R. Bielecki, J. Jakubowski, M. Niewegłowski, Construction and Simulation of Generalized Multivariate Hawkes Processes, *Stochastic Models*, (2022) online
[4] T.R. Bielecki, J. Jakubowski, M. Niewegłowski, Markovianization of Multivariate Hawkes processes, *preprint*,

Optimal Transport methods for calibration of pricing models in mathematical finance

Jan Obłój
University of Oxford
e-mail: jan.obloj@maths.ox.ac.uk

We discuss the problem of calibrating a pricing model in mathematical finance to market data. We show that the problem can be formulated as a semimartingale optimal transport problem under a finite number of discrete constraints. A classical mimicking argument allows us to restrict ourselves to Markovian dynamics and yields a PDE formulation along with its dual counterpart. The solution, a calibrated diffusion process, can be obtained by optimising over solutions to the Hamilton-Jacobi-Bellman equation arising from the dual formulation. The approach, first proposed in [2,3], draws parallels with the one introduced in [1] in the context of entropy minimisation. The method is tested on both simulated

data and market data. We first address the joint calibration problem of SPX options and VIX options or futures, a problem known to be difficult. Numerical examples show that our approach can handle the data well and produces a LV model which is accurately calibrated to SPX options, VIX options and VIX futures simultaneously. We then consider joint calibration to interest rates products and SPX options. Time permitting, we will also discuss the prospect of extending the methodology to cover American options. The talk is based on [4] and ongoing works with Ivan Guo, Benjamin Joseph and Gregoire Loeper.

[1] M. Avellaneda, C. Friedman , R. Holmes and D. Samperi, Calibrating volatility surfaces via relative-entropy minimization, *Applied Mathematical Finance*, 1997, 4(1), 37–64

[2] I. Guo and G. Loeper, Path dependent optimal transport and model calibration on exotic derivatives, *Ann. Appl. Prob.*, 2021, 31(3), 1232–1263

[3] I. Guo, G. Loeper and S. Wang, Calibration of local-stochastic volatility models by optimal transport, *Mathematical Finance*, 2022, 32(1), 46–77

[4] I. Guo, G. Loeper, J. Obłój and S. Wang, Joint Modeling and Calibration of SPX and VIX by Optimal Transport, *SIAM J. on Financial Math.*, 2022, 13(1), 1–31

Stopping games with asymmetric information

Jan Palczewski
University of Leeds
e-mail: J.Palczewski@leeds.ac.uk

This talk is concerned with zero-sum Dynkin (stopping) games in continuous time with partial and asymmetric information when stopping payoffs of players are general càdlàg adapted processes. There are no particular assumptions on the structure of information available to the players. I will show that the game has a saddle point (hence a value) in randomised strategies and provide a martingale characterisation of this saddle point. The talk is based on [1] and ongoing research with Tiziano De Angelis and Jacob Smith.

[1] De Angelis, T., Merkulov, N., Palczewski, J., On the value of non-Markovian Dynkin games with partial and asymmetric information, *The Annals of Applied Probability*, 2022, 32(3), 1774–1813

Cancellable American options under negative discounting

Zbigniew Palmowski
Wrocław University of Science and Technology
e-mail: zbigniew.palmowski@pwr.edu.pl

Cancellable American options, also known as game options or Israeli options, are American-style derivatives which give the writer the right to terminate the contract for a fixed penalty. I will talk about perpetual cancellable American put options on an asset whose dynamics follow exponential spectrally negative Lévy process. The price and optimal strategies of the buyer and the writer can be deduced from the solution of a corresponding Dynkin game. The new feature of the model is the negative interest rate which brings in difficulties (the payoff

grows exponentially fast in time) and interesting strategies. We employ fully probabilistic arguments to argue the existence of the value and of the optimal strategies and characterise explicitly their form. We also prove smooth fit at boundaries of stopping sets enabling their numerical identification.
The talk is based on the joint work with Jan Palczewski.

Heat equation with non-homogeneous Dirichlet white noise boundary conditions

Szymon Peszat
Jagiellonian University
e-mail: napeszat@cyf-kr.edu.pl

We study inhomogeneous Dirichlet boundary value problems associated to a linear parabolic equation $\frac{du}{dt} = Au$ with strongly elliptic operator A on bounded and unbounded domains with white noise boundary data. Our main assumption is that the heat kernel of the corresponding homogeneous problem enjoys the Gaussian type estimates taking into account the distance to the boundary. Under mild assumptions about the domain, we show that A generates a C_0-semigroup in weighted L^p-spaces where the weight is a proper power of the distance from the boundary. We also prove some smoothing properties and exponential stability of the semigroup. Finally, we reformulate the Cauchy-Dirichlet problem with white noise boundary data as an evolution equation in the weighted space and prove the existence of Markovian solutions. The talk is based on a joint work with Ben Goldys (Sydney), see [3].

[1] G. Da Prato and J. Zabczyk, Evolution equations with white-noise boundary conditions, *Stochastics Stochastics Rep.*, 1993, 42, 167–182
[2] Z. Brzeźniak, B. Goldys, S. Peszat, F. Russo, Second order PDEs with Dirichlet white noise boundary conditions, *J. Evol. Equ.* , 2015, 15, 1–26
[3] B. Goldys, S. Peszat, Heat equation with non-homogeneous Dirichlet white noise boundary conditions, *J. Differential Equations*, 2023, 362, 382–437
[4] N.V. Krylov, The heat equation in $L^q((0,T), L^p)$-spaces with weights, *SIAM J. Math. Anal.*, 2001, 32, 1117–1141
[5] N. Lindemulder and M. Veraar, The heat equation with rough boundary conditions and holomorphic functional calculus, *J. Differential Equations*, 2020, 269, 5832–5899

Generative modeling for time series via Schrödinger bridge

Huyên Pham
Université Paris Cité
e-mail: pham@lpsm.paris

We propose a novel generative model for time series based on Schrödinger bridge (SB) approach. This consists in the entropic interpolation via optimal transport between a reference probability measure on path space and a target measure consistent with the joint data distribution of the time series. The solution is characterized by a stochastic differential equation on finite horizon with a path-dependent drift function, hence respecting the temporal dynamics of the time series distribution. We can estimate the drift function from data samples either

by kernel regression methods or with LSTM neural networks, and the simulation of the SB diffusion yields new synthetic data samples of the time series. The performance of our generative model is evaluated through a series of numerical experiments. First, we test with a toy autoregressive model, a GARCH Model, and the example of fractional Brownian motion, and measure the accuracy of our algorithm with marginal and temporal dependencies metrics. Next, we use our SB generated synthetic samples for the application to deep hedging on real-data sets. Finally, we illustrate the SB approach in high dimension for generating sequence of images.

Based on joint work with M. Hamdouche and P. Henry-Labordère.

Discrete-time risk sensitive portfolio optimisation with proportional transaction costs

Marcin Pitera
Jagiellonian University
e-mail: marcin.pitera@uj.edu.pl

We discuss a discrete-time risk sensitive portfolio optimisation over a long time horizon with proportional transaction costs. We show that within the log-return i.i.d. framework the solution to a suitable Bellman equation exists under minimal assumptions and can be used to characterise the optimal strategies for both risk-averse and risk-seeking cases. Moreover, using numerical examples, we show how a Bellman equation analysis can be used to construct or refine optimal trading strategies in the presence of transaction costs.

[1] M. Pitera, Ł. Stettner, Discrete-time risk sensitive portfolio optimization with proportional transaction costs, *preprint*, arXiv:2201.02828

On the ergodic properties of certain stochastic models with memory

Miklós Rásonyi
Rényi Institute and Eötvös Loránd University, Budapest
e-mail: rasonyi@renyi.hu

Recently, there has been intensified interest in models of financial markets driven by fractional Brownian motion. In particular, "rough" volatility models have been considered. We discuss, based on [1], results about the existence of invariant probabilities for such models in the theoretical framework of Markov chains in random environments.

We also discuss, in a discrete-time setting, stochastic stability of random evolutions driven by coloured noises, reviewing results of [2].

[1] B. Gerencsér and M. Rásonyi, Invariant measures for multidimensional fractional stochastic volatility models, *Stochastics and PDEs*, 2022, 10, 1132–1164
[2] M. Varvenne, Rate of convergence to equilibrium for discrete-time stochastic dynamics with memory, *Bernoulli*, 2019, 25, 3234–3275

Long-time asymptotics of the value function in nonlinear stopping problems

Andrzej Rozkosz
Nicolaus Copernicus University
e-mail: rozkosz@mat.umk.pl

We consider quite general class of nonlinear stopping problems associated with Markov processes. We will give conditions ensuring that the value functions V_T of these problems with finite horizon T converge as $T \to \infty$ to the value functions V of the corresponding problems with infinite horizon. We will also show that in many interesting cases one can estimate the rate of convergence. Our results will be illustrated by examples. The talk is based on [1].

[1] T. Klimsiak, A. Rozkosz, Long-time asymptotic behaviour of the value function in nonlinear stopping problems, *ALEA, Lat. Am. Probab. Math. Stat.*, 2021, 19, 1133–1160

On the Separation of Estimation and Control in Risk Sensitive Investment Problems under Incomplete Observation

Wolfgang Runggaldier
Padova University
e-mail: runggal@math.unipd.it

Stochastic control problems under partial observations are often approached by first estimating the unobserved quantities and then solving the problem with these quantities replaced by their estimates. However, in so doing, one looses in general full optimality and, if optimality is still retained, one says that a separation property holds. On the other hand, a standard way is to consider the so called separated problem with the state given by the conditional/filter distribution that in general is an infinite-dimensional object. A further possibility, especially for problems of the risk sensitive type, is to consider the so-called Modified Zakai Equation (MZE) approach (Nagai, Peng) where the MZE is a SPDE such that the objective function of the partially observed problem can be expressed in terms of its solution.

Here, considering an investment problem of the risk sensitive type under partial observation, we start from the underlying assumption that a finite dimensional filter exists, or can be approximated by such a filter and

i) show full equivalence of the original and separated problems,

ii) derive a criterion to identify situations where a separation property holds,

iii) extend the applicability of the MZE approach and show that, starting form a finite-dimensional filter, the MZE reduces to a deterministic PDE.

(Joint with Sébastien Lleo)

Hedging under volatility uncertainty and convex transaction costs

Agnieszka Rygiel
Cracow University of Economics
e-mail: rygiela@uek.krakow.pl

We study discrete-time (super) hedging of contingent claims in uncertain volatility market model. This model captures the case of a financial market which is organized so that the costs are convex with respect to transacted amounts. For options with convex payoff profile, we show that the infimum replicating cost is equal to the one calculated in a binomial model with appropriate parameters.

Convergence of Optimal Strategies in a Multivariate Financial Market with Knightian Uncertainty on the Drift

Jörn Sass
RPTU Kaiserslautern-Landau
e-mail: joern.sass@rptu.de

It is a by now classical observation that in a financial market simple portfolio strategies can outperform more sophisticatedly optimized portfolio strategies. For example, in a one-period setting, the equal weight or $1/N$-strategy often provides more stable results than mean-variance-optimal strategies. This is due to the fact that a reliable estimation of mean returns is not possible for volatile financial assets. Pflug, Pichler and Wozabel (2012) gave a rigorous explanation for this observation by showing that for increasing uncertainty on the means the equal weight strategy becomes optimal in a mean-variance setting – due to its robustness.

We extend this result to continuous-time strategies in a multivariate Black-Scholes type market. To this end we investigate how optimal trading strategies for maximizing expected utility of terminal wealth under CRRA utility behave when we have Knightian uncertainty on the drift, meaning that the only information is that the drift parameter lies in a so-called uncertainty set. The investor takes this into account by considering that the worst possible drift within this set may occur. In this setting we prove a minimax theorem which enables us to find the worst-case drift and the optimal robust strategy quite explicitly. This again allows us to derive the limits when uncertainty increases and hence to show that a uniform strategy is asymptotically optimal.

We also discuss the extension to a financial market with a stochastic drift process, combining the worst-case approach with filtering techniques. This leads to local optimization problems, and the resulting optimal strategy needs to be updated continuously in time. We carry over the minimax theorem to these local optimization problems and derive the optimal strategy. In this setting we show how an ellipsoidal uncertainty set can be defined based on filtering techniques and we demonstrate that investors need to choose a robust strategy to be able to profit from additional information. The talk is based on joint work with Dorothee Westphal.

BSDEs with mean reflection and two constraints

Leszek Słomiński
Nicolaus Copernicus University
e-mail: leszeks@mat.umk.pl

We will discuss the problem of existence and uniqueness of solutions of backward stochastic differential equations with two constraints and a minimality condition depending on the law of the solution (and not on its paths). Connections between expectations of solutions and value functions of appropriately defined deterministic optimization problems will be given. The talk is based on [1,2].

[1] A. Falkowski, L. Słomiński, Mean reflected stochastic differential equations with two constraints, *Stoch. Process. Appl.*, 2021, 141, 172–196
[2] A. Falkowski, L. Słomiński, Backward stochastic differential equations with mean reflection and two constraints, *Bull. Sci. Math.*, 2022, 176, 103117

Optimal certainty equivalent control of the average cost

Łukasz Stettner
Institute of Mathematics PAS
e-mail: stettner@impan.pl

For a given controlled Markov process (X_t) with transition operator $P^{a_n}(X_n, \cdot)$, at generic time n with control a_n taking values in a compact space A and measurable with respect to available observation, we want to minimize the following functional

$$\limsup_{n\to\infty} \frac{1}{n} U^{-1}\left(E\left[U(\sum_{i=0}^{n-1} c(X_i, a_i))\right]\right), \tag{1}$$

where U is a utility function. Such problem was studied in [1] for finite state space using the Arrow-Pratt sensitivity function for so called regular utilities (for which there exist a limit at ∞ of the Arrow-Pratt function). The case with general state space and general utility is studied in the preprint [2] and these results will be presented.

[1] R. Cavazos-Cadena, D. Hernández-Hernàndez, A characterization of the Optimal Certainty Equivalent of the Average Cost via the Arrow-Pratt Sensitivity Function, *Mathematics of Oprations Research*, 2016, 41(1), 224–235
[2] Ł. Stettner, Optimal certainty equivalent control of the average cost, *preprint to be finished soon*,

Multilateral decisions with stopping strategies

Krzysztof Szajowski
Wroclaw University of Science and Technology
e-mail: Krzysztof.Szajowski@pwr.edu.pl

The independent problem of the choice of the moment of action in a bilateral problem appeared in Dynkin (1969) (see [1]). The formulation of a stopping

game assumes that there are two or more players who take turns making decisions, and each player has the option to either continue or stop the game at their turn. The players must make their decisions based on the information they have about the game's state and their opponent's behavior. The players may have different objectives, and the outcome of the game depends on the sequence of decisions made by the players.

The assumption that makes the stopping game problem interesting is that the players have incomplete information about each other's preferences or strategies. This means that each player must take into account the possibility that their opponent will stop the game at any given turn, and the outcome of the game may depend on the player's ability to anticipate and react to their opponent's actions. The players may also face uncertainty about the future rewards or payoffs associated with continuing or stopping the game, which further complicates the decision-making process.

Therefore, already in the formulation of the model, there is a need to take into account differences between players in terms of access to information or willingness to cooperate. Their individual preferences must be included in the definition of a **rational conflict resolution**. This is because these differences can significantly affect the players' strategies and the outcome of the game.

There are different ways to incorporate these factors into the model of a stopping game. For example, one approach is to use game theory, which provides a framework for analyzing strategic interactions between rational decision-makers. Game theory can help identify the equilibrium strategies of the players, which are the strategies that are *optimal* given the other players' strategies.

Overall, the key to successfully modeling and solving stopping game problems is to carefully consider the players' information, preferences, and incentives, and to use appropriate analytical tools and techniques to derive optimal strategies and outcomes.

Also, it should not be forgotten that not all multilateral multiple stop (state selection) tasks are in fact games that can be unambiguously identified with stopping games. A detailed analysis of the Dynkin(1969) game shows that this is a bilateral sequential problem for which an auxiliary stopping game can be created (cf. [2], [3], [4]). Examples of applied solutions in this area are the subject of this report.

[1] E. B. Dynkin, Game variant of a problem on optimal stopping, *Soviet Math. Dokl.*, 1969, 185(1), 270–274
[2] E. G. Enns, E. Z. Ferenstein, On a multiperson time-sequential game with priorities., *Sequential Anal.*, 1987, 6(3):239–256. ISSN 0747-4946. doi: 10.1080/07474948708836129.
[3] E. Presman, I. Sonin, Equilibrium points in a game related to the best choice problem., *Theory Probab. Appl.*, 1975, 20:770–781.
[4] T. Radzik, K. Szajowski, Sequential games with random priority., *Sequential Analysis*, 1990, 9(4), 361–377. ISSN 15324176; 07474946. doi: 10.1080/07474949008836218.

On exponential functional of Brownian motion at a random time

Maciej Wiśniewolski
University of Warsaw
e-mail: wisniewolski@mimuw.edu.pl

We provide the asymptotic bounds on the log-Laplace transform of the exponential functional A of Brownian motion at deterministic time and then at the

random time being the first visit of 0 after fixed deterministic t. Using excursion theory, in the case of A taken at random time we find the explicit form of its distribution in terms of the local time at 1 of a Bessel process with index 0. We provide the connection between this functional and the excursions of a two-dimensional Brownian motion from a hyperplane. It turns out that such excursions are strictly connected with the distribution of a Bessel process with index 0. Finally, we detect the implications of the new results on exponential functional on the small deviations of Gaussian Multiplicative Chaos on Euclidean ball. The talk is based on joint work with Jacek Jakubowski [1].

[1] J. Jakubowski, M. Wiśniewolski, On exponential functional of Brownian motion at a random time, *Preprint*, 2023

Stochastic Models and Optimal Control of Epidemics Under Partial Information

Ralf Wunderlich
Brandenburg University of Technology
e-mail: ralf.wunderlich@b-tu.de

Mathematical models of epidemics such as the COVID-19 pandemics often use compartmental models dividing the population into several compartments. Based on a microscopic setting describing the temporal evolution of the subpopulation sizes in the compartments by stochastic counting processes one can derive macroscopic models for large populations describing the average behavior by associated ODEs such as the celebrated SIR model. Further, diffusion approximations allow to address fluctuations from the average and to describe the state dynamics also for smaller populations by stochastic differential equations (SDE).

Usually not all of the state variables are directly observable and we are facing the so-called "dark figure" problem addressing for example the unknown number of asymptomatic and non-detected infections. Such not directly observable states are problematic if it comes to the computation of characteristics of the epidemic such as the effective reproduction rate and the prevalence of the infection within the population. Further, the management and containment of epidemics relying on solutions of (stochastic) optimal control problems and the associated feedback controls need observations of the current state as input.

The estimation of unobservable states based on records of the observable states leads to a non-standard filtering problem for partially observable stochastic models. We adopt the extended Kalman filter approach coping with nonlinearities in the state dynamics and the state-dependent diffusion coefficients in the SDEs. Based on these filtering results we study a stochastic optimal control problem under partial information arising in the cost-optimal management of epidemics.

This is joint work with Florent Ouabo Kamkumo and Ibrahim Mbouandi Njiasse (Cottbus).

A New Computational Approach for Nonlinear Filtering

George Yin
University of Connecticut
e-mail: gyin@uconn.edu

Nonlinear filtering is a fundamental problem in control and systems theory. In the 1960s, celebrated results on nonlinear filtering were obtained. Nevertheless, the computational issues for nonlinear filtering remained to be a long-standing and challenging problem. In this talk, in lieu of treating an infinite dimensional problem for obtaining the conditional distribution, or conditional measure, we construct finite-dimensional approximations using deep neural networks for the optimal weights. Two recursions are used in the algorithm. One of them is the approximation of the optimal weight and the other is for approximating the optimal learning rate. [This is a joint work with Q. Zhang, and H. Qian.]

Book presentations "Mathematics of the bond market" by M.Barski J.Zabczyk and "Mathematical control theory", 2nd edition, by J. Zabczyk

Jerzy Zabczyk
Insitute of Mathematics Polish Academy of Sciences
e-mail: jerzy@zabczyk.com

The book "Mathematics of the bond market" [1] is concerned with models in which random factors are represented by Lévy processes. The starting points were 1997 papers by Bjork, Kabanov, Rungaldier [2] and Bjork, Di Masi, Kabanov, Rungaldier [3]. Important contributions were made, in particular, by Filipovic, Eberlein, Jacod, Tappe and Teichman. Some parts of the book are based on the authors publications.

Our first aim was to mathematically characterize those Lévy bond markets which are free of arbitrage. Intuitively a market is arbitrage free if a trader is not able to generate profit without taking risk. The second main concept we analyzed was completeness of the market. Again intuitively, a market is complete if a trader can construct a strategy that reproduces any prespecified financial contract. A useful tool to construct arbitrage-free bond markets are stochastic equations. The stochastic equations which appear here are nonlinear, and sometimes with partial derivatives and nonlocal. They constitute important part of the book. The book consists of four parts: I. Bond Market in Discrete time, II. Fundamentals of stochastic analysis, III. Bond market in continuous time, IV. Stochastic equations in the bond market. An appendix is devoted to Martingale Representation for Jump Lévy processes.

The first edition of "The Mathematical Control Theory. An Introduction" [6] appeared in 1992. The aim was to present, in a concise way, all basic parts of the deterministic control theory. It consisted of four parts: I. Elements of Classical Control Theory, II. Nonlinear control systems, III. Optimal Control, IV. Infinite-dimensional linear systems. The new elements of the second edition

can be described as follows. A new chapter on controllability with vanishing energy, see [7], was added to Part I together with an application to orbital transfer problem [8]. A short proof of the Routh stability criteria is presented. Part II is essentially as before. A detailed proof of the existence and uniqueness of the viscosity solutions of Bellman's equations is added to Part III. Boundary control systems and delayed control systems are discussed in chapter IV. Improved stability results for general hyperbolic systems are presented.

[1] M Barski, J.Zabczyk, Mathematics of the bond market. A Lévy processes approach, *Cambridge University Press*, 2020
[2] T. Bjork, Y. Kabanov, W. Rungaldier, Bond market structure in the presence of of marked point processes, *Mathematical Finance*, 1997, 7, 211–239
[3] T. Bjork, G. Di Massi, Y. Kabanov, W. Rungaldier, Towards a general theory of bond markets, *Finance and Stochastics*, 1997, 1, 141–174
[4] M. Barski, J. Zabczyk, On CIR equations with general factors, *SIAM Journal on Financial Mathemtics*, 2020, 11, 131–147
[5] M. Barski J. Zabczyk, Heath–Jarrow-Morton-Musiela equations with Lévy perturbation, *Journal of Differential Equations*, 2012, 253, 2657–2697
[6] J. Zabczyk, Mathematical control theory. An introduction, *2nd edition*, Birkhauser
2020 [7] E.Priola, J. Zabczyk, Null controllability with vanishing energy, *SIAM J. Control Otim.*, 2003, 41, 1013–1032
[8] M.Shibata, A. Ichikawa, Orbital Rendezvou and Flyaround based on controllability with vanishing energy, *J.Guid. Control Dyn.*, 2007, 30