FROM POISSON BRACKETS
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PREFACE

The Workshop *From Poisson brackets to universal quantum symmetries* was one of the major events of the international research project under the same name (supported by NCN under grant 2012/06/M/ST1/00169), and was hosted at IMPAN from the 18^{th} to the 22^{th} of August 2014. The main aim of the workshop was to gather together experts belonging to different sectors of the vast area of quantization all interested in the role played by symmetries of geometric origin.

Quantum groups are the most important topic related to quantization problems; much of what we know about them today was developed by the Warsaw school. They first arose in the work of Fadeev and his collaborators as a natural object to encode symmetries in quantization of classical integrable symmetries. It was Drinfel'd who clarified the relevant algebraic and geometrical structures, in his address as a recipient of the Fields Medal at the Berkeley ICM Congress in 1984, and it was Woronowicz who developed the necessary tools for a C^*-algebraic approach. The representation theory of quantum groups, as well as the very subtle relation between the algebraic and analytical approaches to them, have been the subject of intense investigation in the last years. Deformation quantization is the study of deformation as noncommutative associative algebras of the algebra of functions on a Poisson manifold, in such a way that the first order limit allows us to reconstruct the Poisson bracket. Since the work by Kontsevich, who showed that formal deformation quantization is always possible, much progress has been achieved, notably various versions of the NC Index Theorems and the solution of so-called Formality Conjectures. In noncommutative geometry the approach is different, in that the starting point is a noncommutative algebra which, in principle, might not be related to a commutative one. By analogy with what happens in the classical case Connes set forth a list of axioms that a noncommutative algebra should satisfy to be able to recover a version of Riemannian geometry, in the form of what is nowadays called a spectral triple. Underlying all such approaches is the "semi-classical" theory, i.e. the geometry of Poisson manifolds, the interest in which was, in fact, much revived after the introduction of Poisson–Lie groups (the semi-classical analogue of quantum groups) and after gaining a better comprehension of Poisson cohomology and homology and characteristic classes therein, again stimulated by analogous work on Hochschild and cyclic homology theories.

Since, in fact, the explicit intention of the whole Research Project is to foster interaction between different mathematical approaches to quantization, the meeting was designed so as to maximize real scientific exchange. A limited number of communications left a vast amount of time for informal discussions. That was indeed appreciated by the participants, judging both by the questions and comments sessions after the talks as well as informal discussions afterwards. Overall 23 people, coming from 8 different countries, participated in the Workshop, as listed below.

The first day was devoted to the role played by universal quantum symmetries, a vast array of techniques to introduce purely quantum effects in the analysis of noncommutative algebras. The second day's topic was, by and large, noncommutative geometry. In more detail, communications concentrated on spectral NC geometry. Interesting new *noncommutative* features appear in a vast class of examples, starting from the classical case of non-commutative tori and extending to various generalizations, which still await a thorough comprehension. The third day focused on the appearance of groupoids and algebroids in various quantization settings. These two algebraic structures have proven to be flexible tools to analyse the relation between geometry and quantization. The fourth day was mainly concerned with some algebraic structures underlying the role of symmetries in quantization. The main topic was the use of Galois objects in proving versions of classical topological decompositions and the role of braidings between algebras. The last day was devoted to the use of symmetries in deformation quantization. From the "softer" geometrical structure of gerbes to the much more rigid setting of quantization of Hamiltonian systems, various group actions can be used as a valuable mathematical tool.

This volume contains a selection of contributions displaying at the same time the vast variety of techniques used in the largely unexplored world of quantum geometry, and the underlying unifying philosophy which was one of the sources of inspiration for this Workshop.

The papers selected for this volume bring significant new ideas, aiming to connect various subjects and be used to foster further research.

Nicola Ciccoli and Andrzej Sitarz

LIST OF PARTICIPANTS

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