TOPOLOGICAL QUANTUM GROUPS
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PREFACE

This volume collects the lectures delivered during the Graduate School *Topological Quantum Groups* which took place between the 28th of June and the 11th of July 2015 in the Mathematical Research and Conference Center in Będlewo (Poland), owned by the Banach Center.

The theory of topological quantum groups is a relatively new and vibrant area of mathematics, having its origins on one hand in the classical concept of group duality and on the other in the need for identifying and studying natural quantum deformations of symmetries appearing in physics. It has been undergoing a rapid development especially since 1980s, with an important Polish contribution represented by the groundbreaking works of Stanisław Lech Woronowicz. In recent years the theory has achieved sufficient maturity to become at the same time an established, independent area of study, and a subject fruitfully interacting with other mathematical fields. More specifically, to mention just a few basic aspects, locally compact quantum groups offer

(i) applications to physics:
- in subfactor theory (quantum field theory): compact/discrete/locally compact quantum groups arise as invariants of subfactors and lead to constructions of new subfactors with interesting properties;
- in noncommutative geometry: quantum groups provide examples of "noncommutative spaces", often with extra structure given for example by Dirac operators;

(ii) a natural framework for noncommutative harmonic analysis:
- locally compact quantum groups form a category that includes classical locally quantum groups as well as their "q-deformations" (the latter appeared in the eighties, introduced in the work of Drinfeld, Jimbo, Woronowicz and others);
- the category of locally quantum groups is closed under taking duals, thus contains a generalization of Pontryagin duality (and Tannaka–Krein duality) and forms a very natural class of objects generalizing abelian groups and stable under duality;

(iii) a rich playground for quantum probability and quantum information theory:
- they lead to the quantum generalization of classical Lévy processes;
- they provide a suitable background to study quantum random walks and related quantum boundaries (connecting also to harmonic analysis mentioned above);

- probability distributions known from free probability and the combinatorics of noncrossing partitions arise naturally in relation with the class of quantum groups called "easy";
- they facilitate constructions of natural examples of quantum channels (i.e. unital, trace preserving, completely positive maps on C^*-algebras) — quantum counterparts of noisy channels of the classical information theory.

The school presented several of these connections, as well as the basic introduction to the theory. It consisted of three introductory lectures delivered by the organisers and the following eight five-hour-long lecture series:

- Teodor Banica, *Compact quantum groups and Hadamard matrices*,
- Michael Brannan, *Approximation properties for operator algebras associated to locally compact quantum groups*,
- Martijn Caspers, *Introduction to locally compact quantum groups*,
- Kenny De Commer, *Actions of compact quantum groups*,
- Sergey Neshveyev, *Compact quantum groups and categories*,
- Zhong-Jin Ruan, *Harmonic analysis and theory of multipliers on locally compact quantum groups*,
- Roland Speicher, *Compact quantum groups and free combinatorics*,
- Reiji Tomatsu, *Boundaries for random walks on discrete classical and quantum groups*.

The school was supported by the Banach Center, Warsaw Center of the Mathematical and Computer Sciences and the NCN (National Centre of Science) grant SONATA BIS 2014/14/E/ST1/00525 of Adam Skalski. It was attended by 49 participants, including 3 organisers and 8 lecturers. They came from Belgium, Canada, France, Germany, India, Iran, Ireland, Italy, Japan, Norway, Poland, Russia, South Korea and USA.

Organisers of the school:
Uwe Franz, Adam Skalski, Piotr M. Sołtan