PHENOMENOLOGICAL APPROACH
TO ALGEBRAIC GEOMETRY
BANACH CENTER PUBLICATIONS, VOLUME 116
INSTITUTE OF MATHEMATICS
POLISH ACADEMY OF SCIENCES
WARSZAWA 2018

PREFACE

In this volume we collect contributions resulting from the Algebraic Geometry Simons Semester miniPAGES in 2016. It contains survey and research articles related to topics of hyperkähler manifolds, $K3$ surfaces, Calabi–Yau manifolds, configurations and arrangements of points, curves, or hyperplanes, asymptotic behaviour of linear systems, Nagata conjecture and Newton–Okounkov bodies.

Overview. Polish Algebraic Geometry mini-Semester, abbreviated miniPAGES, was a research activity lasting for the period of two months from April 18 to June 18, 2016, hosted by the Institute of Mathematics of the Polish Academy of Sciences (IMPAN) in Warsaw. MiniPAGES was a part of the larger research program *Simons Semesters in Banach Center* running from July 1, 2015 to December 31, 2019 as a part of yet larger funding program Targeted Grants for Institutes by Simons Foundation.

The purpose of miniPAGES was to promote algebraic geometry and its interactions with other disciplines of science. It provided excellent training opportunities for young researchers including the occasion for interactions between themselves and the top experts in the area.

The leading topics of this intense program were hyperkähler manifolds, $K3$ surfaces, Fano varieties, Calabi–Yau varieties, linear series and broadly understood positivity. For more details about the semester see `http://minipages.impan.pl`. This volume originates from teaching and research activities held or initiated during miniPAGES.

Organization. The main venue of the mini-semester was the Institute of Mathematics of the Polish Academy of Sciences (IMPAN) in Warsaw. Two weekly seminars took place at this location, the Research Seminar coordinated by Jarosław Buczyński and the Young Researches Seminar coordinated by Justyna Szpond. The institute generously provided the working space for all the participants, so that naturally it was the venue of ample informal discussions, spontaneous lectures and social events. The main senior scientists: Igor Dolgachev, Brian Harbourne, Viacheslav Nikulin and Kieran O'Grady had there their headquarters and attracted numerous students and visitors from all over Poland and abroad.

The Faculty of Mathematics, Informatics, and Mechanics of the Warsaw University hosted another main seminar. It was coordinated by Adrian Langer and Jarosław Wiśniewski. The weekly talks there were divided into an introductory part and the following research talk devoted to contemporary developments of the theory.

There were three major scientific events held during the mini-semester.

Introductory School: Varieties of Calabi–Yau type. The school was organized by Michał Kapustka, Oskar Kędzierski, Lars Halvard Halle and Vladimir Lazić at the Institute of Mathematics of the Polish Academy of Sciences (IMPAN) in the week April 18–22, 2016. Its aim was to introduce the participants to the research area of the mini-semester. Calabi–Yau manifolds, $K3$ surfaces and, more generally, hyperkähler manifolds play an important role in modern algebraic geometry and in mathematical physics. Hyperkähler and Calabi–Yau manifold are defined in terms of their special holonomy groups ($Sp(n)$ and $SU(n)$) and jointly they are called *varieties of Calabi–Yau type*. The two classes coincide in dimension 2 with $K3$ surfaces. The main lecture series were delivered by Viacheslav Nikulin, Kieran O'Grady and Balázs Szendrői.

Workshop: Asymptotic invariants attached to linear series. The workshop was organized by Jarosław Buczyński, Piotr Pokora, Sławomir Rams and Tomasz Szemberg at the Department of Mathematics of the Pedagogical University in Cracow in the week May 17–21, 2016. As asymptotic methods play increasingly a prominent role in algebraic geometry and commutative algebra and a number of new tools and methods has been introduced recently, the purpose of the workshop was to introduce the participants to this circle of ideas and simultaneously to create a venue for new scientific collaborations in this area of algebraic geometry. There were three lecture series held by Brian Harbourne, Alex Küronya and Joaquim Roé. These lectures, in considerably expanded versions, are presented in this volume.

Closing conference: Varieties with trivial canonical bundles. The conference was organized by Paweł Borówka, Sławomir Cynk, Maria Donten-Bury, Grzegorz Kapustka and Sławomir Rams at the Mathematical Research and Conference Center in Będlewo (part of the Banach Center) in the week June 13–17, 2016. It was the concluding event of the mini-semester that brought together almost 70 top specialists and graduate students in the area. They shared their recent results, including results obtained during the mini-semester, in 24 talks. The present volume reports on some of research initiated during the miniPAGES and finished shortly after.

Support. MiniPAGES were generously supported by Simons Foundation under Award Number 346300. Additional support was provided by the

- Ministry of Science and Higher Education of Republic of Poland under a 2015–2019 fund;
- Foundation Compositio Mathematica;
- Banach Center;
- Warsaw Center of Mathematics and Computer Science;
- City of Cracow under the program Cracow Science Conferences.

Organizing such a complex event requires economic resources but these would be useless without human resources. Apart of colleagues already mentioned above, the event obtained considerable support from: Łucja Farnik, Maciej Gałązka, Maks Grab and Joachim Jelisiejew. Scientifically we were advised by Thomas Bauer, Fedor Bogomolov, Ciro Ciliberto, Igor Dolgachev, Mark Gross, Rick Miranda, Piotr Pragacz and Claire Voisin.

As main organizers the editors of the present volume would like to take the opportunity to thank heartily all people who contributed to the great success of the event.

The present volume. This volume contains contributions by participants and speakers of the mini-semester.

Dima Al Tabba and Alessandra Sarti continue study of non-symplectic automorphisms of 2-power order of $K3$ surfaces. They classify order 8 automorphisms with fourth power acting trivially on the Picard group and leaving invariant an elliptic curve.

Gwyn Bellamy, Travis Schedler and Ulrich Thiel study hyperplane arrangements for a symplectic quotient singularity defined as Calogero–Moser hyperplanes through the representation theory of restricted rational Cherednik algebras. In particular they prove that the two definitions give the same arrangements, which explains why in all known examples the second definition gives a union of hyperplanes.

Dominik Burek uses Chen–Ruan orbifold cohomology to give shorter and simpler proofs of a formula for the Hodge numbers of generalized Borcea–Voisin Calabi–Yau threefolds originally constructed by Cattaneo and Garbagnati.

Adam Czapliński and Piotr Pokora prove that four extremal line configurations (Böröczky, s-elliptic, polyhedral and Fermat) share equal asymptotic characteristic numbers (Chern slopes and the asymptotic linear Harbourne constant).

Ljudmila Kamenova surveys finiteness results for deformations of hyperkähler manifolds. She improves her own result on the finiteness of deformation classes of hyperkähler manifolds with fixed Fujiki constant and Beauville–Bogomolov–Fujiki lattice (dropping the assumption of existence of a Lagrangian fibration) and simplifies the proofs of some other results.

Brian Harbourne's lectures revolve around asymptotic invariants of linear systems and homogeneous ideals. He reports about recent developments where arrangements of lines play a prominent and somewhat unexpected role. The lectures cover the Bounded Negativity Conjecture, optimality issues in the containment problem and divisors with unexpected properties.

The lectures of Alex Küronya and Victor Lozovanu focus on geometrically relevant properties of Newton–Okounkov bodies. They contain an outline of the authors' recent joint results. They focus on various positivity properties of divisors on surfaces encoded in the geometry of associated NO-bodies. A great deal of attention is given to singularities of divisors on abelian surfaces and their syzygies.

Joaquim Roé's and Paola Supino's lectures seek their motivations in the long standing open Nagata Conjecture. They report on a series of papers by Ciro Ciliberto and Rick Miranda on the Nagata Conjecture and related statements in the spirit of the Segre–Harbourne–Gimigliano–Hirschowitz Conjecture. They pass then to the geometry of various cones of curves determined by positivity notions in the Néron–Severi space. The Nagata Conjecture has been recently reformulated in the language of rank 1 valuations. This point of view is explored in considerable depth in the notes.

Jarosław Buczyński, Sławomir Cynk and Tomasz Szemberg