

PREFACE

The modern theory of exterior differential systems (EDS) originates with Élie Cartan’s brilliant idea of studying nonlinear partial differential equations through the geometric properties of the set of their “infinitesimal solutions”—otherwise known as “integral elements”. A particularly well-behaved example of an EDS is the contact EDS associated with a contact manifold, whose set of integral elements is a bundle naturally modelled on the Lagrangian Grassmannian, a nonsingular projective algebraic variety and a homogeneous space as well. Moreover, contact EDS’es provide the natural environment for studying second-order nonlinear PDEs.

A Lagrangian Grassmannian is then a natural catalyst of interactions between the general theory of EDS, projective geometry, representation theory, geometric theory of second-order nonlinear PDEs, and integrable systems. The overlapping of these disciplines has been overlooked so far, in spite of its evidence, and the aim of this volume is precisely that of clarifying it. It is based on the *Workshop on Geometry of Lagrangian Grassmannians and Nonlinear PDEs*, held at the Banach Centre in Warsaw, September 5–9, 2016. The workshop consisted of mini-courses and expository seminars, aimed not at presenting the latest research results, but rather at making the basic toolbox of each discipline accessible to the experts in the others, in order to create the necessary synergies to tackle open problems.

Following the spirit of the workshop, the present volume offers a synthetic and effective presentation of the fundamentals of such nontrivial theories like EDS, Lie Sphere Geometry, and Projective Duality, as the authors managed to distill their expertise in a three-hour mini-course. The four mini-courses were delivered during the first three days of the meeting by Gary Jensen, Benjamin McKay, Abraham David Smith, and Francesco Russo. This volume contains also expository and review contributions closely related to the topics of the mini-courses. Open problems and current research topics are underlined throughout the text.

The volume will serve as an efficient introduction for the newcomer, a reference for the experienced researcher, and a document of interaction between the topics covered. Due to the pedagogical character of the contributions, and their tight interconnections, the volume possesses a certain inner structure, allowing it to be used as a textbook for a monographic graduate/postgraduate course. To this end, the editors have prepared the special article *Geometry of Lagrangian Grassmannians and nonlinear PDEs*, containing a gentle introduction to the main topic of the homonymous workshop/volume, a comprehensive bibliography, and key references to the other contributions. The editors hope it

will clarify the common background behind the various papers, thus helping the reader to find his/her own way in the volume.

The workshop was attended by twenty five persons from all over the world. Among them, Jun-Muk Hwang (Korean Institute of Advanced Studies), a master geometer and originator of the concept of varieties of minimal rational tangents, who kindly agreed to give an unscheduled seminar. The editors of the present volume, also organisers of the workshop, wish to thank all the participants to the workshop and the personnel of the Banach Centre, for their efficient and professional contribution to the realisation of the meeting. They acknowledge the financial support of the Banach Centre of the Institute of Mathematics of the Polish Academy of Sciences, of the Warsaw Centre of Mathematics and Computer Sciences, of the Italian project FIR (Futuro in Ricerca) 2013 *Geometria delle equazioni differenziali*, and of the Marie Skłodowska-Curie Action No 654721 “GEOGRAL”. They also express their gratitude to Dmitri V. Alekseevsky, Arman Taghavi-Chabert, Letterio Gatto, Wojciech Kryński, Piotr Mormul, Curtis Porter and Raffaele Vitolo for helping us to shape the volume in its final form.

Jan Gutt, Gianni Manno, Giovanni Moreno