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INTRODUCTION

The Second International Conference *Arithmetic Methods in Mathematical Physics and Biology* (`https://ammpb2.wmi.amu.edu.pl/`) was held on August 5–11, 2018, in the Mathematical Research and Conference Center of the Institute of Mathematics, Polish Academy of Sciences, in Będlewo, Poland. The first conference in this series was held on August 3–8, 2014 (`https://ammpb.wmi.amu.edu.pl/`). The papers from that conference were published in [1]. Both conferences were highly multidisciplinary spanning as different areas of natural sciences as Cryptography, Quantum Computing and Tumor Pathology.

Arithmetic and Algebra form the basis of Mathematics, but have not often found direct applications in natural sciences. Physics has been an exception among natural sciences since the beginning of the era of contemporary research. Mathematics became a formal language for Physics as a golden standard. In fact, the emergence of quantum computing would not be possible without Arithmetic and Algebraic Quantum Mechanics, Operator Theory and very recently the Galois theory. And vice versa, Physics has brought inspiration for numerous mathematical theories. For example, String Theory, Mirror Symmetry and Quantum Field Theory stimulated both advanced geometric and topological research. A unifying theory of Statistical Biological Physics is unlikely. Physical models or equations cannot usually be applied directly to solve problems in biological complexity owing to spatial inhomogeneity of biological objects, systems or phenomena. However, both stochasticity and large numbers of interacting elements are present in almost all biological phenomena regardless of the level of organization. Hence, biological complexity gave an impulse to some mathematical approaches, for example, to population genetics developed by Fisher, Hardy, Wright and others at the beginning of the 20th century or to stochastic partial differential equations of the Fokker–Planck type applied in modelling of gene expression or mutation spreading in a population (reviewed in [5]). The situation has changed qualitatively in the last 30 years. The advanced research on biological complexity resulted in some novel mathematical theorems, such as the contribution of evolutionary biologists to metric spaces in the form of the Dress–Bandelt Split Decomposition that enables polyhedral framework in phylogenetic reconstruction [2]. Further theorems are to be expected soon in the theory of dynamical systems, deterministic chaos and fractal geometry. Mathematical methods are also applied successfully to investigate a relationship between genotype, phenotype, and a weak emergence of cancer.

The interest in arithmetic and algebraic methods within the research community of natural scientists continues to grow as exemplified by our interdisciplinary conferences in 2014 and in 2018. Indeed, both mathematical disciplines have always been sources of lively ideas that fuelled the development of natural sciences, starting from a choice of a "0–1" numerical system for the basis of computations and ending in cellular automata that iterate some simple algebraic rules resulting in complex geometric self-similar structures (fractals). Second, natural sciences evolve on their own. They become not only more and more interdisciplinary by adopting approaches that were already applied in the other disciplines, such as the application of fractal geometry in linguistics [1], of cellular automata in modeling of biochemical reaction in the Krebs cycle [4], or of the Galois theory in quantum computing [1]. Natural sciences also approach the emerging problems far beyond the traditional areas of interests, such as image analysis in Tumor Pathology, big data analysis in Oncology, time series analysis in Cardiology, or quantum computing; phenomena that may be best understood in the framework of physical and biological complexity. Those emerging problems require, however, an application of some utterly novel mathematical tools or computer algorithms, such as graph neural networks or convolutional neural networks for the accurate classification of malignant tumors. Third, the lack of a common mathematical language is one of the most important obstacles in Applied Sciences. Therefore, the co-operation between mathematicians and researchers representing different natural sciences is unavoidable. In particular, one cannot imagine a further development in Oncology without the objective, quantitative methods of image analysis in both Tumor Pathology and Radiology that determine the choice of therapy.

The purpose of the current conference was further development of arithmetic methods in specific areas, such as Cryptography, Mathematical Physics and Quantitative Tumor Pathology, in part introduced during the first Conference in 2014. Speakers presented their approaches to those specific issues and showed a number of unique findings briefly summarized below. All lectures delivered during the conference and submitted as articles to the Proceedings clearly demonstrated a deep interplay between natural sciences and mathematical sciences. The Scientific Committee and the organizers of that Conference strongly believe that it is important to bring awareness of the importance of arithmetic and algebraic methods for contemporary research in physical and biological complexity.

The contents of the manuscripts, in the same order as they appear in the volume, are presented below.

1. Helmut Ahammer with collaborators submitted a paper concerning heart rate variability. The authors investigate the influence of optical drifts and power supply noise, both of which may degrade the recorded signals, on nonlinear parameters used for quantitating variabilities.

2. Investigation of Diophantine equations and their solutions has always been very important in Cryptography. Stefan Barańczuk, in his paper, investigates the Diophantine equation $(x^2-1)(y^2-1)=(z^2-1)^2$ from the point of view of arithmetic geometry. He shows that it defines — up to a trivial factor — an elliptic surface. Then he compares his result on integer solutions to the results of Schinzel and Sierpiński.

3. Another very important tool in Cryptography is discrete logarithm. In her paper Dorota Blinkiewicz investigates the multiple base discrete logarithm problem in G_m^n and E_d^n. She considers the local to global principle in the n-th power of G_m and the n-th power of the CM elliptic curve $E_d : y^2 = x^3 - d^2x$. In both cases, for $n > 1$, she gives examples where the local to global multiple base discrete logarithm problem fails.

4. Herbert Jelinek and his team submitted two research papers. In the first paper they apply the box-counting fractal analysis method in three dimensions to the x, y, z data obtained during a Y-Balance Test (YBT) and the 3-dimensional fractal dimension (3DFd) of CoM dispersion. They observe that 3-dimensional analysis provides a more complete indication of complexity compared to current 2-dimensional movement variability measures.

5. In the second paper they studied multifractal analysis of proprioception and postural stability. They observed that the multifractal analysis of the dynamic changes in center of mass showed a typical multifractal spectrum with $D(q)$ values suggesting differences between the control and both taping conditions. Postural sway during the YBT confirmed that complex sensorimotor feedback is required as characterized by the multifractal spectra with or without tape. Both kinesiology tape and rigid tape showed improvement in postural stability during the YBT, which may be associated with improved proprioception.

6. The first paper of Piotr Krasoń and Jan Milewski concerns an eigenproblem for inverted one-dimensional harmonic oscillator. They find a complete description for the eigenproblem in $C^{\infty}(\mathbb{R})$ with eigenfunctions described in terms of the confluent hypergeometric functions and the spectrum equal to $\mathbb{C}$. However, the only states that have physical significance are those which lie in $L^2(\mathbb{R})$ and correspond to the real eigenvalues. The authors identify those states by three methods: analytical, comparison (by means of a certain unitary operator) and involving rigged Hilbert spaces.

7. The second paper concerns the arithmetic of the Heisenberg ring and cyclic group actions. The authors compute in some new cases the cardinalities of the fibers of certain natural fibrations that appear in the analysis of the configuration space of the Heisenberg ring. This is done by means of certain cyclic group actions on some subsets of restricted partitions.

8. The aim of the paper of Levente Simon and Anna Soós is the investigation of the fixed point limits of growing self-similar networks. The results are interpreted on the generalized form of the scale-free network. The paper is based on weighted graph edit distances defined on these networks and on sets of growing network sequences with the corresponding parametrized weighted graph edit distances. The authors show that the iterated function systems corresponding to the self-similar networks have unique fixed points.

9. Fractal geometry became a language for research on biological complexity in many areas of medical sciences. One of the lively aspects has been studied by Nebojša T. Milošević and Velicko Vranes. The authors attempt to establish how different neuron groups are related to the function of dentate nucleus. They studied 272 microscopic images of neurons using computer algorithms. Each image was characterized by eleven morphometric parameters: six of them were Euclidean and five monofractal. In addition, five parameters quantified neuron size, three quantified shapes, and three quantified complexity of neurons. The results put the anatomical/topological classification in a serious doubt since large subsets of border and central neurons differed only in three monofractal parameters, which quantified their two properties. In addition, cluster analysis classified large multipolar neurons into three groups, which differed significantly in relation to all parameters used in that study. Those findings represent an important contribution in both mathematical and experimental neuroscience.

10. Rasha Abu-Eid and co-workers used tools of fractal geometry to investigate changes in the spatial distribution, size and shape of collagen type IV between normal, dysplastic and neoplastic tissues. The authors observed a pattern, where the particle size was significantly smaller in low- and high-grade dysplasia compared to normal oral tissues. However, there was a significant increase in the size for these particles in cancer cases. Collagen Type IV can thus be applied in oral cancer pathology to determine its aggressiveness.

11. Yoichi Uetake reviews some recent developments in linking theories of systems, scattering and operators, focusing on the notion of spectral interpretation. The author discusses the relation between infinite-dimensional linear system theory and Lax–Phillips scattering theory. He suggests some connections of these theories with arithmetic in terms of a spectral interpretation.

12. Przemysław Waliszewski presented results of his theoretical considerations as well as a clinical and statistical analysis in a set of prostate carcinomas. He found a novel relationship between the coefficient of cellular expansion and the Fibonacci constant. He demonstrated that the Fibonacci constant is a limit for the ratio of the integer dimension of the complementary Euclidean space, in which growth, proliferation and self-organization of cells into tissue structures of the higher order, such as glands, occur and the global spatial capacity fractal dimension characterizing the spatial distribution of cells. He also found the corresponding value of the coefficient of cellular expansion that determines limits for those processes in a normal prostate tissue. Furthermore, he showed that the local fractal dimensions greater than 1.700 indicate cancer cells with the enhanced metastatic potential since those cells do not interact with each other, and, therefore, may metastasize easily. That finding is associated with the probability of formation of distant metastases in prostate carcinomas. The algorithms applied in that study can be used as routine methods in Quantitative Tumor Pathology. In addition, Waliszewski hypothesizes the existence of a relationship between entropy and both global and local fractal dimensions. Intercellular interactions generate dynamics limited by the Mandelbrot set. The hypothesis is in concert with the observation that there is a relationship between a planar shape filled by a polynomial Julia set and a measure of maximal entropy [3].

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