(3) *There exists a p-ordered sequence of homogeneous generators of I $u_1, \ldots, u_p, \ldots, u_n$, $h(u_i) = h_i$, such that the Tate resolution X of A is equal to*

$$X = F_2X = A\langle T_1, \ldots, T_n, dT_i = u_i\rangle\langle S_{p+1}, \ldots, S_n, dS_j = u_j^{h_j-1}T_j\rangle$$

with $\partial(T_i) = \partial(u_i)$, $w(T_i) = 1$, $\partial(S_j) = h_j\partial(u_j)$, $w(S_j) = 2$.

(4) *There exists a p-ordered sequence $u_1, \ldots, u_p, \ldots, u_n$ of homogeneous generators of the ideal I, $h(u_i) = h_i$, such that*

$$H_1(B) = H_2(B) = 0\,,$$

where $B = A\langle T_1, \ldots, T_n, dT_i = u_i\rangle\langle S_{p+1}, \ldots, S_n, dS_j = u_j^{h_j-1}T_j\rangle$.

Proof. The equivalence of (1) and (2) is contained in Proposition 1.14. Implication (2) $\Rightarrow$ 3) follows from Corollary 2.8 and (3) $\Rightarrow$ (4) is obvious. Finally, implication (4) $\Rightarrow$ (3) holds in virtue of Proposition 2.6.

2.11. Remark. We do not know if the above theorem is true for graded R-algebras which are not finitely generated.

The following example shows that not every graded R-algebra A with $X = F_2X$ is h-regular, i.e. satisfies one of the equivalent conditions from Theorem 2.10.

2.12. Example. Let k be a field and let $A = k[X, Y]/(XY)$, where $\partial(X) = \partial(Y) = 2$. Since XY is a non-zero divisor in $k[X, Y]$, we have $X = F_2X$ by [2], IV, § 2 Theorem 1. At the same time one can easily prove that A is not an h-regular graded k-algebra.

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Absolute retracts as factors of normed linear spaces

by

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Abstract. It is shown that if X is a (complete) AR($\mathfrak{M}$)-space, then, for a suitable (complete) normed linear space E, $X\times E$ and E are homeomorphic. This implies that l_2-manifolds are characterized as separable, complete, l_2-stable ANR($\mathfrak{M}$)'s.

In this paper we deal with products of absolute retracts and normed linear spaces. We shall show that any absolute retract is a factor of a normed linear space, i.e. if $X \in$ AR($\mathfrak{M}$), then there is a normed space E such that $X\times E$ is homeomorphic to E. We also show that normed spaces E of a more special type (e.g. all infinite-dimensional Hilbert spaces) have the property that each retract of E is also a factor of E.

The paper is a sequel to [28] and we shall use some terms and notation of [28]. In particular, we shall say that a retraction r of a metric space (Y, d) is *regular*, if r is continuous and

(*) for every $\varepsilon > 0$ the set $\{y \in Y\colon d(r(y), y) \geqslant \varepsilon\}$ is of positive d-distance from $r(Y)$.

The main result of [28] was:

Theorem 0. *Let r be a regular retraction of a normed linear space $(E, \|\ \|)$ and let $X = r(E)$. Then $X\times\sum_{l_1}E \cong \sum_{l_1}E$, and if, moreover, X is complete in the norm $\|\ \|$, then also $X\times\prod_{l_1}E \cong \prod_{l_1}E$. Here, $\prod_{l_1}E = \{(t_i) \in E^\infty\colon \sum_{i\geqslant 1}\|t_i\| < \infty\}$ and $\sum_{l_1}E = \{(t_i) \in E^\infty\colon t_i = 0$ for almost all $i\}$, both spaces being equipped with the norm $|||(t_i)||| = \sum_{i\geqslant 1}\|t_i\|$.*

To apply this theorem, we examine here regular retractions more accurately. Unexpectedly enough, it appears (see Section 2) that on any absolute retract X there is an admissible metric ϱ with the property that every closed isometric embedding of (X, ϱ) into a metric space Y maps X

onto a regular retract of Y. Combining this with the Arens–Eells Embedding Theorem, we obtain in Section 3 the above-mentioned results concerning products of absolute retracts and normed spaces, and we use them next in discussing the following subjects: (i) products of ANR's and normed spaces and the identifying of l_2-manifolds (Section 4); (ii) products and factors of simplicial complexes with metric topology (Section 5), and (iii) tubular neighbourhoods of Z-embeddings of ANR's (Section 6). Section 1 is of preliminary character; for convenience in references we include in it the results of [5], [20] and [27] which we need in this paper.

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1. Preliminaries. Throughout the paper we shall denote by R the set of real numbers and by N the set of positive integers. The elements of the extended reals $[-\infty, \infty]$, as well as the $[-\infty, \infty]$-valued functions, will be denoted by the Greek letters $\varepsilon, \delta, \lambda, \mu$, while positive integers will be denoted by i, j, k, n. The closed intervals of both $[-\infty, \infty]$ and N will be denoted by $[\varepsilon, \delta]$; thus, $\{\lambda: \lambda \in [\varepsilon, \delta]\}$ is the "real" interval, while by $\{i: i \in [\varepsilon, \delta]\}$ we mean the intersection of this interval with N. We assume $\infty + \infty = \infty$.

If not stated otherwise, by "retraction" we mean "continuous retraction". We shall say "r is a regular retraction of Y" instead of "r is a regular retraction of (Y, d)" if the metric d is clear from the context. The notation and definitions concerning Absolute Retracts and Absolute Neighbourhood Retracts are those of [7].

Given a metrizable space X we denote by $\mathrm{Metr}(X)$ the set of all metrics on X which induce the topology of X; elements of $\mathrm{Metr}(X)$ will be called *admissible metrics* for X. If $\varrho \in \mathrm{Metr}(X)$ and $A \subset X$, then for $x \in X$ we let $\mathrm{dist}_\varrho(x, A) = \inf\{\varrho(x, a): a \in A\}$. By $\mathrm{dens}(X)$ we mean the least cardinality of dense subsets of X, $\mathrm{card}(A)$ denotes the cardinality of a set A, and we shall write $(X, A_1, \ldots, A_n) \cong (Y, B_1, \ldots, B_n)$ to indicate that there is a homeomorphism $f\colon X \xrightarrow{\text{onto}} Y$ with $f(A_i) = B_i$ for $i \in [1, n]$. $C(T, X)$ denotes the space of all continuous maps from T to X.

Let $(E, \|\ \|)$ be a normed linear space. By E^∞ we denote the countable Cartesian power of E, considered in the product topology, and we let $\sum E = \{(t_i) \in E^\infty: t_i = 0 \text{ for almost all } i\}$ be the topological subspace of E^∞ (not to be confused with the space $\sum_{l_1} E$ defined in the formulation of Theorem 0). The convex hull of a set $A \subset E$ will be denoted by $\mathrm{conv}\, A$.

For the sake of convenience let us introduce the following notation:

$E_1(A) = l_2(A)$, the Hilbert space of square-summable real functions on A;

$E_2(A) = l_2(A) \times \sum R$;

$E_3(A) = l_2^f(A) \overset{\mathrm{df}}{=} \{\lambda \in l_2(A): \lambda(a) = 0 \text{ for almost all } a \in A\}$;

$E_4(A) = l_2^f(A) \times Q$, where $Q = [-1, 1]^\infty$ is the Hilbert cube.

To those spaces there correspond some classes of metric spaces, which we shall denote as follows:

$\mathfrak{M}_1 =$ the class of complete-metrizable spaces;

$\mathfrak{M}_2 =$ the class of metric spaces which are countable unions of closed, complete-metrizable sets;

$\mathfrak{M}_3 =$ the class of metric spaces which are countable unions of locally compact, locally finite-dimensional sets;

$\mathfrak{M}_4 =$ the class of metric spaces which are countable unions of locally compact sets.

The spaces $l_2(N)$ and $l_2^f(N)$ will also be denoted by l_2 and l_2^f respectively. Below we list some results describing the properties of the spaces $E_i(A)$, $i \in [1, 4]$:

1.1. *Let $i \in [1, 4]$ and let A be an arbitrary set. Then a space X with $\mathrm{dens}(X) \leqslant \mathrm{card}(A)$ admits a closed embedding into $E_i(A)$ if and only if $X \in \mathfrak{M}_i$.*

Proof. The assertion is well known for $i = 1$ (see [20], proof of Corollary 2.4 or [6], p. 606); for the proof in the case $i \in [2, 4]$ see [27], § 7.

1.2. *For any infinite set A we have:*

(a) $\sum R \cong l_2^f$ *and* $E_2(A) \cong l_2(A) \times l_2^f$;

(b) $l_2^f \times Q \cong E_\sigma$ *and (hence)* $E_4(A) \cong l_2^f(A) \times E_\sigma$, *where* $E_\sigma = \{\lambda \in l_2: \sum i^2\lambda(i)^2 < \infty\}$ *is the subspace of* l_2.

In particular, each of the spaces $E_i(A)$, $i \in [1, 4]$, is homeomorphic to a pre-Hilbert space.

For a proof see [5], § 5, and [27], § 7.

1.3. *Let X be a space with $X \times E_2(A) \cong E_2(A)$ and let $i \in \{3, 4\}$. If $X \in \mathfrak{M}_i$ then $X \times E_i(A) \cong E_i(A)$.*

Proof. Assume first $i = 3$. It follows from [27] that $X \times l_2^f(A) \times l_2^f$ and $l_2^f(A) \times l_2^f$ are $\mathcal{K}$-absorbing sets in $X \times l_2(A) \times l_2^f$ and $l_2(A) \times l_2^f$ respectively, where $\mathcal{K}$ denotes the family of all locally compact finite-dimensional subsets of the space in question. Since $l_2(A) \times l_2^f \cong E_2(A)$ (see 1.2), it follows from our assumption and from the general properties of absorbing sets that $X \times l_2^f(A) \times l_2^f \cong l_2^f(A) \times l_2^f$.

In the case $i = 4$ the proof is similar (replace $l_2^f(A) \times l_2^f$ by $E_4(A)$ and $\mathcal{K}$ by the family of all locally compact sets).

1.4. *Let $(E, \|\ \|)$ be a normed linear space and let A be a set of cardinality $\mathrm{dens}(E)$.*

(a) *If E is a Banach space then there exists a space F such that* $E\times F\cong l_2(A)$;

(b) *If E is a countable union of its closed subsets, each being complete in the norm $\|\;\|$, then* $E\times E_2(A)\cong E_2(A)$.

Proof. Part (a) is shown in [20], pp. 28–29 (cf. [4], p. 266) and part (b) in [27], § 7.

2. Regular retractions and regular metrics. Let X be an $\mathrm{AR}(\mathfrak{M})$-space. In this section we shall be concerned with the question whether X can be embedded in a normed linear space as its regular retract. Let us make some introductory remarks. Assume for simplicity that X is a closed subset of a normed linear space $(E,\|\;\|)$. By the definition of $\mathrm{AR}(\mathfrak{M})$-spaces, there is in this case a retraction $r\colon E\xrightarrow{\text{onto}}X$; unfortunately, r need not be regular in any translation-invariant metric of E. For example, if E_0 denotes the Euclidean plane and X_0 is the set

$$\{1\}\times[0,1]\cup\{(x_1,0)\colon x_1\geqslant 1\}\cup\{(x_1,1/x_1)\colon x_1\geqslant 1\},$$

then it is easy to observe that there is no regular retraction of E_0 onto X_0. Obviously X_0 is homeomorphic to $X_1=\{(x_1,0)\colon x_1\in R\}$, which is a regular retract of E_0. To get the required re-embedding in this special case we had to "push apart" some "parts" of X_0; however, it is not clear how these parts should be defined in the general situation (¹). Are they, for instance, simply neighbourhoods of distinct points of the remainder of a properly defined completion of X? It is so for $(E,X)=(E_0,X_0)$ but when drawing other examples it appears that one should rather depend on the interrelation of a fixed retraction $r\colon E\xrightarrow{\text{onto}}X$ and the affine structure of E: e.g., in any of the simplest examples one has to "push apart" any two sequences (a_n) and (b_n) of points of X for which we have both $\inf\limits_n\|a_n-b_n\|=0$ and $\inf\{\mathrm{diam}_{\|\;\|}\,r([a_n,b_n])\colon n\in N\}>0$ ($[a_n,b_n]$ denotes $\{\lambda a_n+(1-\lambda)b_n\colon \lambda\in[0,1]\}$). Extending this idea, we shall obtain the required re-embedding in two steps: we construct first a metric ϱ on X such that for no sequence (A_n) of subsets of X we have $\inf\{\mathrm{diam}_\varrho A_n\colon n\in N\}=0$ while $\inf\{\mathrm{diam}_\varrho r(\mathrm{conv}A_n)\colon n\in N\}>0$, and we then show that any closed isometric embedding of (X,ϱ) into a normed space $(F,|||\;|||)$ maps X onto a regular retract of F.

The detailed proofs run as follows:

2.1. Proposition. *Let $r\colon K\xrightarrow{\text{onto}}X$ be a retraction of a convex subset K of a locally convex linear metric space. Then, for every $\varrho_0\in\mathrm{Metr}(X)$, there*

(¹) We say that a map $h\colon(X,\varrho)\to(Y,d)$ pushes the sets $A,B\subset X$ apart, if $\mathrm{dist}_\varrho(A,B)=0$ while $\mathrm{dist}_d(h(A),h(B))>0$.

is a $\varrho\in\mathrm{Metr}(X)$ and a function $\varepsilon\colon[0,\infty]\to[0,\infty]$ such that: (1) $\lim\limits_{\delta\to 0}\varepsilon(\delta)=0$, (2) *for every $A\subset X$ the ϱ-diameter of $r(\mathrm{conv}A)$ does not exceed $\varepsilon(\mathrm{diam}_\varrho A)$, and* (3) $\varrho\geqslant\varrho_0$.

The $\varrho-\varrho_0$ will be obtained as a sum of a countable number of metrics, each built inductively on the basis of the following

Sublemma. *Let K, X and r be as above and let $\varrho_i\in\mathrm{Metr}(X)$ be fixed Then there exists a $\varrho_{i+1}\in\mathrm{Metr}(X)$ such that*

(*i) *For any $n\in N$ and $A\subset X$ with $\mathrm{diam}_{\varrho_{i+1}}A\leqslant 2^{-n}$, we have*

$$\mathrm{diam}_{\varrho_i}r(\mathrm{conv}A)\leqslant n^{-1}2^{-n}.$$

Proof. Given $n\in N$, let $\mathcal{U}_n$ be a cover of X consisting of (relatively) open sets which are so small that $\mathrm{diam}_{\varrho_i}r(\mathrm{conv}\,U)\leqslant n^{-1}2^{-n}$ for all $U\in\mathcal{U}_n$. By a lemma of E. Michael, there are metrics d_n, $n\in N$, such that every set of d_n-diameter less than 1 is contained in a member of $\mathcal{U}_n$ ([23], p. 165). We let

$$\varrho_{i+1}(x_1,x_2)=\sum_{n=1}^{\infty}\min\big(d_n(x_1,x_2),2^{-n+1}\big),\qquad x_1,x_2\in X.$$

Proof of Proposition 2.1. Starting from the metric ϱ_0, construct inductively a sequence $\varrho_1,\varrho_2,\ldots$ of admissible metrics on X satisfying for every $i\geqslant 0$ condition (*i), and define $\varrho\in\mathrm{Metr}(X)$ by the formula

$$\varrho(x_1,x_2)=\varrho_0(x_1,x_2)+\sum_{i=1}^{\infty}\min\big(\varrho_i(x_1,x_2),2^{-i+1}\big),\qquad x_1,x_2\in X.$$

If $A\subset X$ is a set with $\mathrm{diam}_\varrho A\leqslant 2^{-n}$, then for all $i\in[1,n]$ we have $\mathrm{diam}_{\varrho_i}A\leqslant 2^{-n}$. This gives $\mathrm{diam}_{\varrho_i}r(\mathrm{conv}A)\leqslant n^{-1}2^{-n}$ for $i\in[0,n-1]$ and, consequently, $\mathrm{diam}_\varrho r(\mathrm{conv}A)\leqslant n\cdot n^{-1}\cdot 2^{-n}+\sum\limits_{i\geqslant n}2^{-i+1}=5\cdot 2^{-n}$. Thus, the metric ϱ and the function ε defined by

$$\varepsilon(\delta)=10\delta\ \text{if}\ \delta\in[0,1/2]\quad\text{and}\quad\varepsilon(\delta)=\infty\ \text{if}\ \delta>1/2$$

satisfy the required conditions.

By a uniform embedding of a metric space (X,ϱ) into another one (Y,d) we shall mean here any embedding $h\colon X\to Y$ such that both h and h^{-1} are uniformly continuous when considered as maps between the metric spaces (X,ϱ) and $(h(X),d)$.

2.2. Proposition. *Let K be a convex set in a linear metric space, let $r\colon K\xrightarrow{\text{onto}}X\subset K$ be a retraction, and let $\varrho\in\mathrm{Metr}(X)$ and a function $\varepsilon\colon(0,\infty]\to(0,\infty]$ satisfy conditions* (1) *and* (2) *of Theorem* 2.1. *Then, for* *every closed uniform embedding h of (X, ϱ) into a metric space (X, d), there is a regular retraction of (Y, d) onto $h(X)$.*

Proof. Let $((\lambda_U, z_U))_{U\in\mathfrak{U}}$ be a Dugundji system for $(Y, h(X), d)$, i.e. we assume that $(\lambda_U)_{U\in\mathfrak{U}}$ is a (continuous and locally finite) partition of unity on $Y\setminus h(X)$ and, for every $U\in\mathfrak{U}$, z_U is a point of $h(X)$ such that

(D) if $y\in Y$ satisfies $\lambda_U(y)\neq 0$, then $d(y, z_U)\leqslant 4\operatorname{dist}_d(y, h(X))$

(for the construction of systems of the required type see [8], p. 188).

We define a retraction $q\colon Y\xrightarrow{\text{onto}} h(X)$ by the formula

$$q(y)=\begin{cases} y & \text{if} \quad y\in h(X),\\ hr\Big(\sum\limits_{U\in\mathfrak{U}}\lambda_U(y)\cdot h^{-1}(z_U)\Big) & \text{if} \quad y\in Y\setminus h(X).\end{cases}$$

Obviously, q is continuous on the open set $Y\setminus h(X)$. Thus, the proof will be completed if we show that q satisfies condition (*) (for q will then also be a continuous map).

For $n=1,2$, let $\varepsilon_n\colon (0,\infty]\to(0,\infty]$ be a function such that $\lim\limits_{\delta\to 0}\varepsilon_n(\delta)=0$ and

$$d\big(h(x_1), h(x_2)\big)\leqslant\varepsilon_1\big(\varrho(x_1, x_2)\big) \quad\text{and}\quad \varrho\big(h^{-1}(y_1), h^{-1}(y_2)\big)\leqslant\varepsilon_2\big(d(y_1, y_2)\big)$$

for all $x_1, x_2\in X$ and $y_1, y_2\in Y$. Without loss of generality assume that ε_1, ε_2 and ε are non-decreasing functions. Fix for a moment $y\in Y$, let $\mathfrak{U}_0$ denote the finite set $\{U\in\mathfrak{U}\colon \lambda_U(y)\neq 0\}$, and choose $V\in\mathfrak{U}_0$. Setting $\delta=\delta(y)=\operatorname{dist}_d(y, h(X))$, we have $d(y, z_U)\leqslant 4\delta$ for all $U\in\mathfrak{U}_0$, whence $\operatorname{diam}_d\{z_U\colon U\in\mathfrak{U}_0\}\leqslant 8\delta$ and $\operatorname{diam}_\varrho\{h^{-1}(z_U)\colon U\in\mathfrak{U}_0\}\leqslant\varepsilon_2(8\delta)$. This gives $\operatorname{diam}_\varrho r(\operatorname{conv}\{h^{-1}(z_U)\colon U\in\mathfrak{U}_0\})\leqslant\varepsilon\circ\varepsilon_2(8\delta)$ and, in particular,

$$\varrho\big(h^{-1}q(y), h^{-1}(z_V)\big)\leqslant\varepsilon\circ\varepsilon_2(8\delta).$$

Letting $\varepsilon_0(\mu)=4\mu+\varepsilon_1\circ\varepsilon\circ\varepsilon_2(8\mu)$, we get $d\big(q(y), z_V\big)\leqslant\varepsilon_1\circ\varepsilon\circ\varepsilon_2(8\delta)$ and $d\big(q(y), y\big)\leqslant\varepsilon_0(\delta)=\varepsilon_0\big(\delta(y)\big)$. Since this inequality holds true for all $y\in Y$ and since $\lim\limits_{\mu\to 0}\varepsilon_0(\mu)=0$, the assertion is proved.

Let us say that a metric ϱ defined on an $\mathrm{AR}(\mathfrak{M})$-space X is a *regular metric* for X if $\varrho\in\operatorname{Metr}(X)$ and, given any closed uniform embedding of (X, ϱ) into a metric space Y, the image of X is a regular retract of Y. Using the fact that every metric space admits a closed embedding into a convex subset of a normed linear space (Kuratowski, Eilenberg and Wojdysławski), we infer from Propositions 2.1 and 2.2:

2.3. Theorem. *If X is an $\mathrm{AR}(\mathfrak{M})$-space, then for every $\varrho_0\in\operatorname{Metr}(X)$ there exists a regular metric ϱ for X such that $\varrho\geqslant\varrho_0$.*

The applications of 2.3 discussed in the subsequent sections depend on the following theorem of R. Arens and J. Eells:

(AE) *Every metric space (X, ϱ) admits an isometric embedding h into a Banach space, such that the set $h(X)$ is both linearly independent and closed in its linear span.*

This theorem, which generalizes earlier results of Kuratowski, Kunugui and Eilenberg–Wojdysławski, was established in [3]; for some shorter proofs see [24] or [30]. Obviously, (AE) and 2.3 imply a positive answer to the question stated at the beginning of this section.

3. Absolute retracts as factors of normed spaces. Theorems 0 and 2.3 combined with the theorem of Arens and Eells yield:

3.1. Theorem. *Given $X\in\mathrm{AR}(\mathfrak{M})$ there is a normed linear space E such that $X\times E\cong E$ and* $\operatorname{dens}(E)=\operatorname{dens}(X)$.

Unfortunately, 3.1 does not give any information on the topological type of the normed space whose factor is X. The remainder of the section will be devoted to obtaining some results in this direction, at least for absolute retracts belonging to one of the classes $\mathfrak{M}_i$, $i\in[1,4]$, discussed in Section 1.

3.2. Theorem. *Let $i\in[1,4]$. If X is an $\mathrm{AR}(\mathfrak{M})$-space with $X\in\mathfrak{M}_i$, then $X\times E_i(A)\cong E_i(A)$, where A is a set of cardinality* $\operatorname{dens}(X)$.

Proof. We shall consider separately the cases $i=1$, $i=2$ and $i\in\{3,4\}$.

(a) $i=1$. Let $\varrho_0\in\operatorname{Metr}(X)$ be a complete metric on X, let $\varrho\in\operatorname{Metr}(X)$ be a regular metric with $\varrho\geqslant\varrho_0$ and let $h\colon X\to L$ be an isometric embedding of (X, ϱ) into a Banach space L of density character equal to that of X. Then ϱ is a complete metric, and therefore h is a closed embedding. By Theorem 0 we get $X\times E\cong E$, where E denotes the Banach space $\prod_{l_1}L$. Thus $X\times E\times F\cong E\times F$ for every space F, and $X\times l_2(A)\cong l_2(A)$ by 1.4 (a).

(b) $i=2$. By 1.1 and 1.2 (a) we can consider X as a closed subset of $l_2^f\oplus l_2(A)$. Let $\|\ \|$ be the norm of $l_2^f\oplus l_2(A)$ and let ϱ_0 denote the metric induced by $\|\ \|$ on X; further let ϱ be a regular metric for X such that $\varrho\geqslant\varrho_0$, and let h be an isometric embedding of (X, ϱ) into a Banach space F with $\operatorname{dens}(F)=\operatorname{dens}(X)$. It is easy to see that

$$x\mapsto\big(h(x), x\big)\in F\oplus l_2^f\oplus l_2(A)$$

is a closed uniform embedding of (X, ϱ) into the normed space $E=F\oplus l_2^f\oplus l_2(A)$. Applying Theorem 0, we get $X\times\sum_{l_1}E\cong\sum_{l_1}E$, which combined with 1.4(b) gives $X\times E_2(A)\cong X\times\sum_{l_1}E\times E_2(A)=\sum_{l_1}E\times\times E_2(A)\cong E_2(A)$.

(c) $i \in \{3, 4\}$. We have $\mathfrak{M}_i \subset \mathfrak{M}_2$ and therefore $X \times E_2(A) \cong E_2(A)$ by the case (b) above. Thus the assertion follows 1.3 (²)

For separable spaces the formulation of the case $i = 4$ of Theorem 3.2 can be simplified:

3.3. Corollary. *If $X \in \mathrm{AR}(\mathfrak{M})$ is a countable union of compact sets, then $X \times E_\sigma \cong E_\sigma$, where $E_\sigma = \{\lambda \in l_2: \sum_{i \geqslant 1} i^2 \lambda(i)^2 < \infty\}$ is the subspace of l_2.*

Proof. By 1.2(b) the space $Q \times l_2^f$ is homeomorphic to E_σ.

Theorem 3.2 can be equivalently stated in the following form

3.4. Corollary. *Let E be a space of the form $E_i(A)$, where $i \in [1, 4]$ and A is an infinite set. Then, for every retract X of E, we have $X \times E \cong E$.*

Question: Does every linear metric space E have the property that each retract of E is a factor of both E^∞ and $\sum E$?

4. Identifying of manifolds. By a cone over a metric space X we mean the topological space $(CX, \mathcal{C}) = (X \times (0, 1] \cup \{0\}, \mathcal{C})$, where $\mathcal{C}$ is the topology generated by open subsets of $X \times (0, 1]$ and sets $\{0\} \cup \cup (0, 1/n) \times X$, $n \in N$.

4.1. Lemma. *If E is a linear metric space such that either E is locally convex or $E \cong E^\infty$ or $E \cong \sum E$, then CE is homeomorphic to a retract of $E \times R$.*

Proof. By results of Henderson ([11], [12, p. 322]), the open cone $CE \setminus E \times \{1\}$ is homeomorphic to $E \times R$. Since CE can be embedded as a retract of $CE \setminus E \times \{1\}$, the result follows.

The following results correspond to 3.1 and 3.2.

4.2. Theorem. *If X is an $\mathrm{ANR}(\mathfrak{M})$-space, then there exists a normed linear space E such that $\mathrm{dens}(E) = \mathrm{dens}(X)$ and $X \times E$ is homeomorphic to an open set $U \subset E$ with $(E, E \setminus U) \cong (CX \times E, \{0\} \times E)$.*

Proof. By (AE), one can consider X as a neighbourhood retract of a normed linear space F with $\mathrm{dens}(F) = \mathrm{dens}(X)$. As is easy to see, CX is a retract of CF (cf. [16], proof on p. 43) and therefore, by Lemma 4.1, CX can be treated as a retract of $F \times R$. Thus there is a normed linear space E_1 such that $\mathrm{dens}(E_1) = \mathrm{dens}(X)$ and $CX \times E_1 \cong E_1$. We set $E = l_2 \times E_1$ and $U = h\big((CX \setminus \{0\}) \times E\big)$, where $h\colon CX \times E \xrightarrow{\text{onto}} E$ is a homeomorphism. Then $U \cong X \times (0, 1] \times E \cong X \times \big((0, 1] \times l_2\big) \times E_1$; since $(0, 1] \times l_2 \cong l_2$ (this result belongs to V. L. Klee and follows also directly from Theorem 3.2), we have $U \cong X \times l_2 \times E_1 \cong X \times E$, which completes the proof.

(²) In the separable case the use of 1.3 can be avoided. (Follow the proof of (b) and use results of Klee [18, p. 190] and of Bessaga-Pełczyński [5, pp. 176–178], cf. [28, p. 82].)

4.3. Theorem. *Let $i \in [1, 4]$, let $X \in \mathfrak{M}_i$, and let A be a set of cardinality $\mathrm{dens}(X)$. If $X \in \mathrm{ANR}(\mathfrak{M})$, then $X \times E_i(A)$ is homeomorphic to an open set $U \subset E_i(A)$ such that $\big(E_i(A), E_i(A) \setminus U\big) \cong \big(CX \times E_i(A), \{0\} \times E_i(A)\big)$.*

The proof is analogous to that of Theorem 4.2.

Theorem 4.3 implies the following version of a special case of Henderson's [12] Open Embedding Theorem:

4.4. Corollary. *Let E be as in 4.3 and let X be a paracompact connected manifold modelled on E. Then X is homeomorphic to an open set $U \subset E$ such that $(E, E \setminus U) \cong (CX \times E, \{0\} \times E)$.*

Proof. By [15], Theorem 6, X admits a closed embedding into E, and by results of Anderson and Schori [2], [26] the spaces $X \times E$ and X are homeomorphic (³). Using this, we obtain the assertion from 4.3 and a result of [25], p. 3, which states that $X \in \mathrm{ANR}(\mathfrak{M})$.

Also, we obtain the following characterization of certain infinite-dimensional manifolds:

4.5. Proposition. *Let X be a metric space, let A be a set of cardinality $\mathrm{dens}(X)$ and let $i \in [1, 4]$. Then, X is an $E_i(A)$-manifold if and only if X is $E_i(A)$-stable (i.e. $X \times E_i(A) \cong X$) and $X \in \mathfrak{M}_i \cap \mathrm{ANR}(\mathfrak{M})$.*

Proof. 1.1 and the proof of 4.4 combine to show that every $E_i(A)$-manifold is an $E_i(A)$-stable space belonging to $\mathfrak{M}_i \cap \mathrm{ANR}(\mathfrak{M})$. The reverse implication follows from 4.4.

Below, two applications of Proposition 4.6 to identifying l_2-manifolds are given; they depend on "stability theorems" for spaces of continuous mappings, due to R. Geoghegan and J. Keesling.

4.6. Corollary. *Let X be a separable complete $\mathrm{ANR}(\mathfrak{M})$-space, $X_1, \ldots, X_n$ its $\mathrm{ANR}(\mathfrak{M})$-subsets of type G_δ, T a compact space and $T_1, \ldots, T_n \subset T$ disjoint closed sets. If U is a cone-patch (in sense of [10]) for T such that $U \subset T_1$ or $U \cap (T_1 \cup \ldots \cup T_n) = \emptyset$ then the space $\{f \in C(T, X)\colon f$ is non-constant on U and $f(T_i) \subset X_i$ for $i \in [1, n]\}$ forms an l_2-manifold when considered in the compact-open topology.*

Proof. By results of Geoghegan (see proofs on pp. 168–169, 171 and 174–175 in [10]) the space in question is l_2-stable. Since it is also an open subset of $Y = \{f \in C(T, X)\colon f(T_i) \subset X_i$ for $i \in [1, n]\}$, the result follows from 4.5 and the fact that Y is a separable $\mathrm{ANR}(\mathfrak{M})$-space (see the proof of VI. 3.1 in [16]) and is a G_δ-subset of the complete space $C(T, X)$.

This easily implies (c.f. examples on p. 168 of [10]).

(³) Cf. Theorem 5 of [15]. The assumptions of the last theorem are satisfied since, by 3.2, 1.1 and the theorem of Dugundji [8, p. 188], we have $E \cong E \times E^\infty$ if E is a Hilbert space and $E \cong E \times \sum E$ otherwise.

4.7. Corollary. *Let $X, X_1, \ldots, X_n$ be as in 4.6, let T be a compact finite-dimensional manifold and let $T_1, \ldots, T_n \subset T$ be disjoint closed sets. Then $\{f \in C(T, X)\colon f$ is non-constant and $f(T_i) \subset X_i$ for $i \in [1, n]\}$ forms an l_2-manifold. In particular, the space of non-constant paths from X_1 to X_2 and the space of non-constant closed curves starting from X_1 form l_2-manifolds* (4).

The second application is:

4.8. Corollary. *Let X be a separable metric space which admits a non-trivial flow, let A and B be subsets of X, and denote by $\mathrm{Auth}_B(X, A)$ the space of all homeomorphism $f\colon X \xrightarrow{\text{onto}} X$ with $f(A) = A$ and $f|B = $ identity ($\mathrm{Auth}_B(X, A)$ is regarded under the compact-open topology). If $\mathrm{Auth}_B(X, A)$ is an $\mathrm{ANR}(\mathfrak{M})$, then it is an l_2-manifold.*

Proof. Apply Proposition 4.5 and a result of Keesling [17], p. 6.

It was shown by R. Luke and W. K. Mason [21], [22], that the space $H(X) = \mathrm{Auth}_{\partial X}(X, X)$ is an $\mathrm{ANR}(\mathfrak{M})$ for every compact $[0,1]^2$-manifold X. This implies:

4.9. Corollary. *If X is a compact 2-dimensional manifold, then $H(X)$ is an l_2-manifold.*

5. Products and factors of metric simplicial complexes.

By a metric simplicial complex we mean here the geometric realization X of a simplicial complex (say, K), endowed with the metric topology of X ([16], p. 99). We shall say that the metric simplicial complex X is *induced* by K and, since only simplicial complexes are considered here, we shall sometimes write "metric complex" instead of "metric simplicial complex".

The following is an extension of a result of J. E. West [32].

5.1. Proposition. *If X is a metric simplicial complex and A is a set of cardinality $\mathrm{dens}(X)$, then $X \times l_2^f(A)$ is an $l_2^f(A)$-manifold and $X \times l_2(A) \times \sum R$ is an $l_2(A) \times \sum R$-manifold.*

Proof. By its definition, X is a closed subset of $l_2^f(A)$ and it is well known that $X \in \mathrm{ANR}(\mathfrak{M})$ ([16], p. 106). Thus $X \in \mathfrak{M}_3 \subset \mathfrak{M}_2$ and the assertion follows from Theorem 4.4.

We note that the density character of the metric complex induced by a connected simplicial complex K is equal to the supremum of the cardinalities of simplices belonging to $\mathrm{st}(v)$, the supremum being taken over all vertices v of K.

(4) The assertion is also true if we omit the words "non-constant". This results from the following fact, which can be established by using 4.6 and a technique of Cutler: If $X \in \mathrm{ANR}(\mathfrak{M})$ and there is a Z-set A in X (see Section 6 for the definition) such that $X \setminus A$ is a l_2-manifold, then X itself is an l_2-manifold. A proof will appear in author's note "Concerning Z-sets in ANR's and characterization of l_2-manifolds".

5.2. Proposition. *Let X be a metric simplicial complex and let A be a set of cardinality $\mathrm{dens}(X)$. If X is complete-metrizable, then $X \times l_2(A)$ is an $l_2(A)$-manifold.*

Proof. Apply Theorem 4.5.

It is clear that a metric complex induced by a simplicial complex K is complete-metrizable iff for every vertex v of K there is no infinite, strictly increasing sequence of simplices belonging to $\mathrm{st}(v)$ ([16], p. 107); in particular, every locally finite-dimensional metric complex is complete-metrizable. (For locally finite-dimensional complexes the assertion of Proposition 5.2 follows from more general results of J. E. West [31].)

Theorem 4.5 allows us also to characterize the spaces which stabilize to metric complexes (see [14] for results on spaces stabilizing to countable CW-simplicial complexes).

5.3. Proposition. *Let X be a connected metric space and let A be a set of cardinality $\mathrm{dens}(X)$. Then the following conditions are equivalent*

(a) *X is homeomorphic to a retract of a metric simplicial complex;*

(b) *$X \in \mathrm{ANR}(\mathfrak{M})$ and X is a countable union of locally compact, locally finite-dimensional sets;*

(c) *There are metric simplicial complexes E and U such that $X \times E \cong U$, $E \cong l_2^f(A)$ and U is homeomorphic to an open subset of $l_2^f(A)$.*

Proof. Implication (a)$\Rightarrow$(b) follows from [16], p. 106, and from the fact that for every metric simplicial complex the differences between its successive finite-dimensional skeletons are locally compact and finite-dimensional.

(b)$\Rightarrow$(c). By Theorem 4.5 the product $X \times l_2^f(A)$ is homeomorphic to an open subset of $l_2^f(A)$. Since the space $l_2^f(A)$ is homeomorphic to a metric simplicial complex [15], and since every open subset of a metric complex is homeomorphic to another metric complex, the implication is proved.

Implication (c)$\Rightarrow$(a) is trivial.

6. Tubular neighbourhoods with infinite-dimensional fibre.

Let $h\colon X \to Y$ be an embedding. By a *trivial tubular neighbourhood* of h we mean a triple $(E, \bar{h}, U)$, where E is a linear metric space, U is an open subset of Y, and $\bar{h}\colon X \times E \xrightarrow{\text{onto}} U$ is a homeomorphism such that $\bar{h}(x, 0) = h(x)$ for all $x \in X$. The space E will be called the *fibre* of the tubular neighbourhood, and, allowing a lack of precision, we shall sometimes say that h or U itself form a tubular neighbourhood of h. If X is a subset of Y and i denotes the inclusion, then we shall say "tubular neighbourhood of X" instead of "tubular neighbourhood of i". It is easy to see that an open set $U \subset Y$ is a trivial tubular neighbourhood of an embedding $h\colon X \to Y$ if and only if it is a trivial tubular neighbourhood of the set $h(X)$.

Let E be a normed linear space. An embedding $h\colon X\to Y$ is said to be *E-deficient* if h is a closed map and there is a homeomorphism $f\colon Y\xrightarrow{\text{onto}} Y\times E$ such that $fh(X)\subset Y\times\{0\}$. Obviously, the space X admits an E-deficient embedding into E iff $E\cong E\times E$ and X is homeomorphic to a closed subset of E.

6.1. LEMMA. *Let E be a locally convex linear metric space such that $E\cong E\times[0,1)$. If X is a space satisfying $CX\times E\cong E$, then every E-deficient embedding $h\colon X\to E$ admits a trivial tubular neighbourhood U (with fibre E) such that $(E, E\setminus U, h(X))\cong(CX\times E, \{0\}\times E, X\times\{1\}\times\{0\})$.*

If the assumption $CX\times E\cong E$ is replaced by $X\times E\cong E$, then E itself is a tubular neighbourhood with fibre E of every E-deficient embedding of X into E.

Proof. Let $E_1 = CX\times E$, $A_1 = X\times\{1\}\times\{0\}$ and $A = h(X)$. The embedding h induces in a natural way the homeomorphism $g\colon A_1\xrightarrow{\text{onto}} A$. Then, E_1 is homeomorphic to E and the sets A_1 and A are E-deficient in E_1 and E respectively (observe that $E\cong E\times E$); thus by a lemma of V. L. Klee ([19], p. 36) there is a homeomorphism $\bar g\colon E_1\xrightarrow{\text{onto}} E$ such that $\bar g|A_1 = g$. We set $U=\bar g(X\times(0,1]\times E)$; it is clear that U is a trivial tubular neighbourhood of h with fibre $E\times(0,1]\cong E$. The proof of the second assertion is left to the reader.

Let I denote the unit interval $[0,1]$. A subset X of a topological space E is said to be a *Z-set* in E iff X is closed in E and, for every n, the set $\{f\in C(I^n, E)\colon f(I^n)\cap X=\emptyset\}$ is dense in $C(I^n, X)$ (5). An embedding $h\colon X\to E$ will be called a *Z-embedding* iff $h(X)$ is a Z-set in E.

We shall use the following lemma:

6.2. LEMMA. *Let E be a locally convex linear metric space such that $E\cong E^\infty$ or $E\cong\sum E$. Then*

(a) $E\cong E\times[0,1)$.

(b) *An embedding into a paracompact E-manifold is E-deficient if and only if it is a Z-embedding.*

For the proof see respectively [5], p. 184 (observe that E has either R^∞ or $\sum R$ as factor) and [27], § 6.

Combining 6.2, 3.2 and 6.1 we get (cf. the footnote (8)):

6.3. THEOREM. *Let E be a space of the form $E_i(A)$, where $i\in[1,4]$ and A is an infinite set, and let h be a Z-embedding into E of a space $X\in\mathrm{ANR}(\mathfrak{M})$. Then there is a trivial tubular neighbourhood U of h such that $(E, E\setminus U, h(X))\cong(CX\times E, \{0\}\times E, X\times\{1\}\times\{0\})$. If, in addition, X is contractible (i.e. $X\in\mathrm{AR}(\mathfrak{M})$), then E itself is a tubular neighbourhood of every Z-embedding $h\colon X\to E$.*

Let us also note that, conversely, if a closed embedding $h\colon X\to Y\in\mathrm{ANR}(\mathfrak{M})$ admits a tubular neighbourhood with infinite-dimensional fibre, then X must be an $\mathrm{ANR}(\mathfrak{M})$ and h must be a Z-embedding. (A proof easily follows from the results of Eells and Kuiper [9]).

6.4. COROLLARY. *Let E be as in Theorem 6.3, let M be a paracompact manifold modelled on E and let $h\colon X\to M$ be a Z-embedding. If $X\in\mathrm{ANR}(\mathfrak{M})$, then h admits a trivial tubular neighbourhood with fibre E.*

Proof. Without loss of generality one can assume that M is a connected open subset of E (see 4.4). By Lemma 6.2(b) there is a homeomorphism $g\colon M\xrightarrow{\text{onto}} M\times E\times R$ such that $gh(X)\subset M\times\{0\}\times\{0\}$. Set

$$g_1(m,t,\lambda)=\big(m,t,\lambda+1/\alpha(m)\big),\qquad (m,t,\lambda)\in M\times E\times R,$$

where $\alpha\colon E\to[0,1]$ is a continuous function with $\alpha^{-1}(0)=E\setminus M$. Then g_1g is a homeomorphism of M onto $M\times E\times R$ and g_1gh is an E-deficient embedding of X into $E\times E\times R\cong E$. By 6.3 there is a tubular neighbourhood U_1 (with fibre E) of g_1gh in $E\times E\times R$, and the standard arguments show that we can assume U_1 to be contained in the open neighbourhood $M\times E\times R$ of $g_1gh(X)$. $g^{-1}(U_1)$ is the required tubular neighbourhood of h in M.

We note also

6.5. COROLLARY. *Let E be a linear metric space which has l_2 as factor (e.g. let E be any infinite-dimensional Fréchet space). Then, every compact ANR-subset of E admits a trivial tubular neighbourhood U with fibre E such that $(E, E\setminus U, X)\cong(CX\times E, \{0\}\times E, X\times\{1\}\times\{0\})$; moreover, E is a trivial tubular neighbourhood with fibre E of each of its compact AR-subsets.*

The proof is the same as that of Lemma 6.1, by using an easy generalization of Klee's homogeneity theorem (cf. [32], p. 262).

Finally we have

6.6. THEOREM. *Let X be an $\mathrm{ANR}(\mathfrak{M})$. Then there is a normed linear space E such that:* (1) $\mathrm{dens}(E)\cong\mathrm{dens}(X)$, (2) *$X$ admits E-deficient embeddings into E, and* (3) *if $h\colon X\to E$ is such an embedding, it has a trivial tubular neighbourhood $(E,\bar h, U)$ with $(E, E\setminus U, h(X))\cong(CX\times E, \{0\}\times\times E, X\times\{1\}\times\{0\})$.*

Proof. By Theorem 3.1 there is a normed linear space F such that $\mathrm{dens}(F)=\mathrm{dens}(X)$ and $CX\times F\cong F$ (cf. the proof of Theorem 4.2). We let $E=\sum_{l_1}F$. Then $E\cong E\times E$ and CX admits a closed embedding into E;

(5) This definition differs from R. D. Anderson's [1] original one; however, if X has a base consisting of homotopy trivial sets, then the Z-sets as defined above coincide with the "sets with Property Z" of [1] and also with the sets which are homotopy negligible (in sense of [9]) in every open subset of X.

thus CX admits also an E-deficient embedding into E. Now apply Lemma 6.1.

Let us note that if $(E, \bar{i}, U)$ is a trivial tubular neighbourhood of a set $X \subset Y$, then there is a (straight deformation) retraction of U onto X which is topologically conjugated to the projection $p_1: X \times E \to X$. From this point of view, Theorem 6.6 can be read as a strengthening of an earlier result of D. W. Henderson ([13], p. 748).

Addendum. The constructions of 2.1 admit easy geometric interpretation, i.e. the proof of 2.1 leads to an F-normed linear space $(H, |||\ |||)$ and to a closed embedding $h: X \to H$ such that the metric

$$\varrho(x_1, x_2) = |||h(x_1) - h(x_2)|||$$

satisfies the assertion of 2.1; moreover, the space H can be built in such a way that $H \cong E^\infty$. This implies that if X is a retract of a locally convex linear metric space E, then $X \times \sum E \cong \sum E$, and if additionally X is complete-metrizable, then $X \times E^\infty \cong E^\infty$. As a corollary one gets that for any Fréchet space F, F^∞ is homeomorphic to a Hilbert space. Details will appear in [29].

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